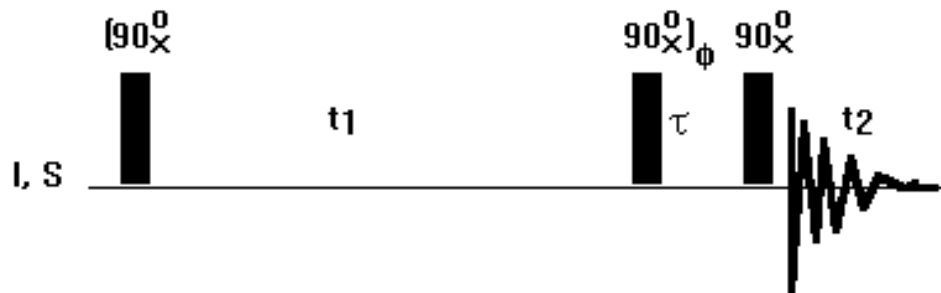


DQF-COSY = Double-Quantum Filtered-CORrelated SpectroscopY

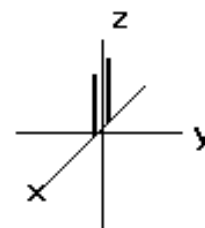
The pulse sequence: $90^\circ_\phi - t_1 - 90^\circ_\phi - \tau - 90^\circ_x - t_2$



Consider: Ω_I , Ω_S and $J_{I,S}$

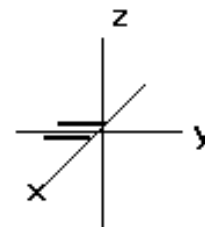
$\sigma[\text{eq.}]$
 $\hat{H} = \pi/2 (\hat{I}_x + \hat{S}_x)$

\mathbf{I}_z and \mathbf{S}_z
 $\downarrow 90^\circ_x$

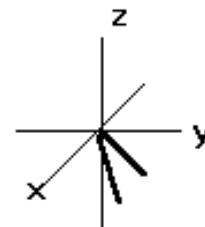


$\sigma[0]$
 $\hat{H} = \hat{I}_z(\Omega_I t_1) + \hat{S}_z(\Omega_S t_1)$

$-\mathbf{I}_y$ and $-\mathbf{S}_y$
 $\downarrow t_1$



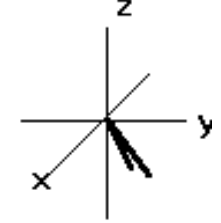
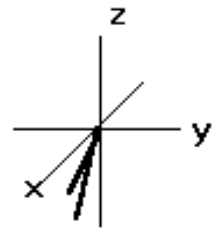
$-\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1)$
 $-\mathbf{S}_y \cos(\Omega_S t_1) + \mathbf{S}_x \sin(\Omega_S t_1)$



$\hat{H} = 2\hat{I}_z \hat{S}_z (J_{I,S} \pi t_1)$

\downarrow

$$\begin{aligned} & -\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1) \\ & -\mathbf{S}_y \cos(\Omega_S t_1) + \mathbf{S}_x \sin(\Omega_S t_1) \end{aligned}$$

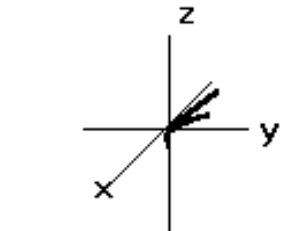


$$\hat{H} = 2\hat{I}_z \check{S}_z (J_{IS} \pi t_1)$$

$$\begin{aligned} & -\mathbf{I}_y \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad + 2\mathbf{I}_x \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & + \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad + 2\mathbf{I}_y \mathbf{S}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & - \mathbf{S}_y \cos(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad + 2\mathbf{S}_x \mathbf{I}_z \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \\ & + \mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad + 2\mathbf{S}_y \mathbf{I}_z \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1) \\ & \quad \downarrow 90^\circ_x \text{ memo. } = \cos(\pi/2) = 0, \sin(\pi/2) = 1 \end{aligned}$$

$$\begin{aligned} & \sigma[t_1] \\ & \hat{H} = \pi/2 (\hat{I}_x + \check{S}_x) \end{aligned}$$

$$\begin{aligned} & -\mathbf{I}_z \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad - 2\mathbf{I}_x \mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & \quad + \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad \quad - 2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & - \mathbf{S}_z \cos(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad - 2\mathbf{S}_x \mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \\ & \quad + \mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad \quad - 2\mathbf{S}_z \mathbf{I}_y \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$



$\sigma[t_1, 0]$
memo.

\mathbf{I}_z and \mathbf{S}_z (z magnetization)

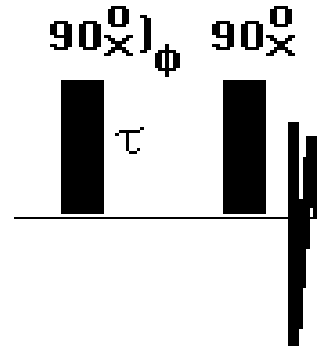
$\mathbf{I}_x \mathbf{S}_y$ and $\mathbf{S}_x \mathbf{I}_y$ (double quant. coherence)

\mathbf{I}_x and \mathbf{S}_x (single quant. coherence)

$\mathbf{I}_z \mathbf{S}_y$ and $\mathbf{S}_z \mathbf{I}_y$ (single quant. coherence)

In DQF-COSY we preserve only double quant. coherences: $\mathbf{I}_x\mathbf{S}_y$ and $\mathbf{S}_x\mathbf{I}_y$
 Therefore only two terms remain:

$$\sigma[t_1,0] \quad \begin{aligned} & -2\mathbf{I}_x\mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & -2\mathbf{S}_x\mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$



In reality $\mathbf{I}_x\mathbf{S}_y$ and $\mathbf{S}_x\mathbf{I}_y$ are still **mixtures** of zero-quantum and double-quantum coherences

memo :

$$\begin{aligned} \mathbf{I}^+ &= \mathbf{I}_x + i\mathbf{I}_y \\ \mathbf{I}^- &= \mathbf{I}_x - i\mathbf{I}_y \\ \mathbf{S}^+ &= \mathbf{S}_x + i\mathbf{S}_y \\ \mathbf{S}^- &= \mathbf{S}_x - i\mathbf{S}_y \end{aligned}$$

consequently: $\frac{1}{2} [\mathbf{I}^+ + \mathbf{I}^-] = \mathbf{I}_x$ and $\frac{1}{2i} [\mathbf{S}^+ - \mathbf{S}^-] = \mathbf{S}_y$

Therefore:

$$-2\mathbf{I}_x\mathbf{S}_y = -2 \left\{ \frac{1}{2} [\mathbf{I}^+ + \mathbf{I}^-] \frac{1}{2i} [\mathbf{S}^+ - \mathbf{S}^-] \right\} = -\frac{1}{2i} \{ \mathbf{I}^+\mathbf{S}^+ - \mathbf{I}^+\mathbf{S}^- + \mathbf{I}^-\mathbf{S}^+ - \mathbf{I}^-\mathbf{S}^- \}$$

only $\mathbf{I}^+\mathbf{S}^+$ and $\mathbf{I}^-\mathbf{S}^-$ are double quant. coh. (or coherence order 2),

$\mathbf{I}^+\mathbf{S}^-$ and $\mathbf{I}^-\mathbf{S}^+$ are zero quant. coh. (or coherence order 0)

In reality *via* phase cycling the zero quantum coherences are removed and thus only

$$-\frac{1}{2i} \{ \mathbf{I}^+\mathbf{S}^+ - \mathbf{I}^-\mathbf{S}^- \} \text{ terms remain.}$$

In conclusion term $-1/(2i) \{(\mathbf{I}^+\mathbf{S}^+) - (\mathbf{I}^-\mathbf{S}^-)\}$ is as follows:

$$\begin{aligned}
 & -1/(2i) [\{(I_x + iI_y)(S_x + iS_y)\} - \{(I_x - iI_y)(S_x - iS_y)\}] = \\
 & -1/(2i) [I_x S_x + iI_y S_x + iI_x S_y - I_y S_y - I_x S_x + iI_y S_x + iI_x S_y + I_y S_y] \\
 & -1/(2i) [iI_y S_x + iI_x S_y + iI_y S_x + iI_x S_y] \\
 & \mathbf{-1/2 [2I_y S_x + 2I_x S_y]}
 \end{aligned}$$

In summary: $-2\mathbf{I}_x \mathbf{S}_y \Rightarrow -1/2 [2\mathbf{I}_y \mathbf{S}_x + 2\mathbf{I}_x \mathbf{S}_y]$ (the 2 true double quant. parts of it)

for the same reason: $-2\mathbf{S}_x \mathbf{I}_y \Rightarrow -1/2 [2\mathbf{I}_x \mathbf{S}_y + 2\mathbf{I}_y \mathbf{S}_x]$

now

$$-2\mathbf{I}_x \mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \Rightarrow -1/2 [2\mathbf{I}_y \mathbf{S}_x + 2\mathbf{I}_x \mathbf{S}_y] \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$-2\mathbf{S}_x \mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \Rightarrow -1/2 [2\mathbf{I}_x \mathbf{S}_y + 2\mathbf{I}_y \mathbf{S}_x] \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1)$$

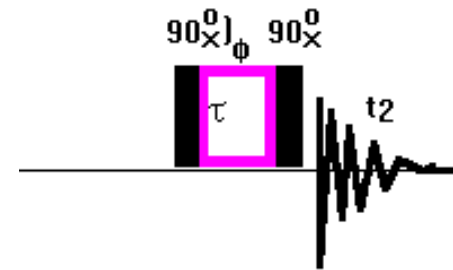
τ is set to be short and thus: - no evolution
- no coupling

$$\begin{aligned}
 & -I_y S_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) - I_x S_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\
 & -I_x S_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) - I_y S_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1)
 \end{aligned}$$

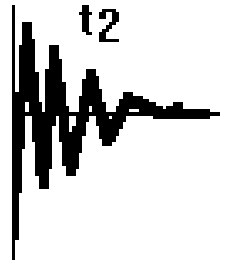
$$\hat{H} = \pi/2 (\hat{I}_x + \check{S}_x)$$

$\downarrow 90^\circ_x$

$$\begin{aligned}
 & -I_z S_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) - I_x S_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\
 & -I_x S_z \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) - I_z S_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1)
 \end{aligned}$$



$$\begin{array}{ll} -\mathbf{I}_z \mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) & -\mathbf{I}_x \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ -\mathbf{I}_x \mathbf{S}_z \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) & -\mathbf{I}_z \mathbf{S}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \end{array}$$



During acquisition (t_2):

$$\hat{H} = \hat{I}_z(\Omega_I t_2) + \check{S}_z(\Omega_I t_2) \text{ and } 2\hat{I}_z \check{S}_z(J_{IS} \pi t_2) \downarrow t_2$$

the $-\mathbf{I}_x \mathbf{S}_z$ term during ACQ \Rightarrow

$$\begin{array}{l} -\mathbf{I}_x \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_I t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{I}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_I t_2) \sin(\pi J_{IS} t_2) \\ +\mathbf{I}_y \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{I}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi J_{IS} t_2) \end{array} \leftarrow$$

the $-\mathbf{I}_z \mathbf{S}_x$ term during ACQ \Rightarrow

$$\begin{array}{l} -\mathbf{I}_z \mathbf{S}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{S}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \sin(\pi J_{IS} t_2) \\ +\mathbf{I}_z \mathbf{S}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{S}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{IS} t_2) \end{array} \leftarrow$$

the $-\mathbf{I}_z \mathbf{S}_x$ term during ACQ \Rightarrow

$$\begin{array}{l} -\mathbf{I}_z \mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \sin(\pi J_{IS} t_2) \\ +\mathbf{S}_y \mathbf{I}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{IS} t_2) \end{array} \leftarrow$$

the $-\mathbf{I}_x \mathbf{S}_z$ term during ACQ \Rightarrow

$$\begin{array}{l} -\mathbf{I}_x \mathbf{S}_z \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_I t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_I t_2) \sin(\pi J_{IS} t_2) \\ +\mathbf{S}_z \mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \cos(\pi J_{IS} t_2) \\ -1/2 \mathbf{I}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi J_{IS} t_2) \end{array} \leftarrow$$

memo 1: receiver on x

therefore only the four x term remain

$$\begin{aligned} & -1/2\mathbf{S}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{IS} t_2) \\ & -1/2\mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{IS} t_2) \\ & -1/2\mathbf{I}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi J_{IS} t_2) \\ & -1/2\mathbf{I}_x \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi J_{IS} t_2) \end{aligned}$$

memo 2:

$$\begin{aligned} \cos(A)\sin(B) &= 1/2[\sin(A+B) - \sin(A-B)] \\ \sin(A)\sin(B) &= 1/2[\cos(A-B) - \cos(A+B)] \end{aligned}$$

therefore

$$\begin{aligned} & +1/8\mathbf{I}_x [+\sin\{(\Omega_I + \pi J_{IS})t_1\} - \sin\{(\Omega_I - \pi J_{IS})t_1\}] [+\cos\{(\Omega_I - \pi J_{IS})t_2\} - \cos\{(\Omega_I + \pi J_{IS})t_2\}] \\ & +1/8\mathbf{I}_x [+\sin\{(\Omega_S + \pi J_{IS})t_1\} - \sin\{(\Omega_S - \pi J_{IS})t_1\}] [+\cos\{(\Omega_I - \pi J_{IS})t_2\} - \cos\{(\Omega_I + \pi J_{IS})t_2\}] \\ & +1/8\mathbf{S}_x [+\sin\{(\Omega_S + \pi J_{IS})t_1\} - \sin\{(\Omega_S - \pi J_{IS})t_1\}] [+\cos\{(\Omega_S - \pi J_{IS})t_2\} - \cos\{(\Omega_S + \pi J_{IS})t_2\}] \\ & +1/8\mathbf{S}_x [+\sin\{(\Omega_I + \pi J_{IS})t_1\} - \sin\{(\Omega_I - \pi J_{IS})t_1\}] [+\cos\{(\Omega_S - \pi J_{IS})t_2\} - \cos\{(\Omega_S + \pi J_{IS})t_2\}] \end{aligned}$$

the following terms can be found

$$\mathbf{I}_x [+ \dots - \dots + \dots - \dots] \text{ at } \Omega_I, \Omega_I$$

$$\mathbf{I}_x [+ \dots - \dots + \dots - \dots] \text{ at } \Omega_S, \Omega_I$$

$$\mathbf{S}_x [+ \dots - \dots + \dots - \dots] \text{ at } \Omega_S, \Omega_S$$

$$\mathbf{S}_x [+ \dots - \dots + \dots - \dots] \text{ at } \Omega_I, \Omega_S$$

if one sets the phase that

sin is absorptive (*a*) in t_1

cos is absorptive (*d*) in t_2

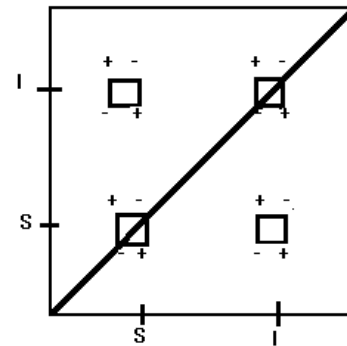
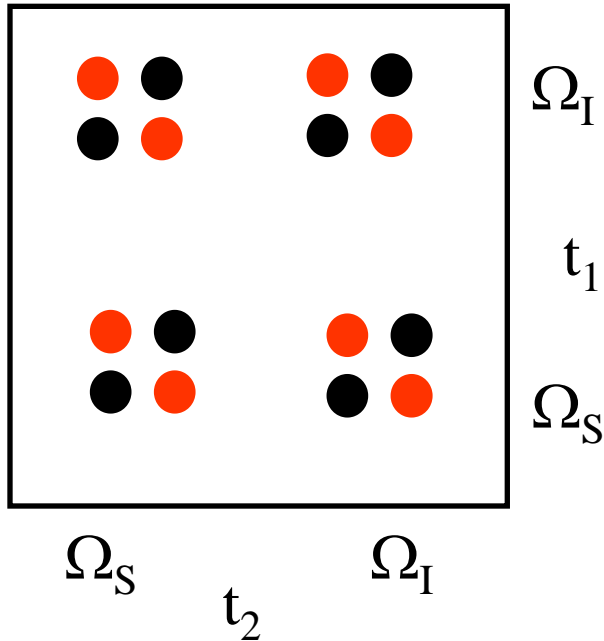
$$\mathbf{I}_x [+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_I, \Omega_I$$

$$\mathbf{I}_x [+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_S, \Omega_I$$

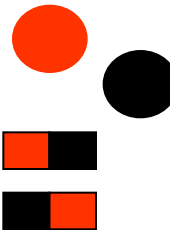
$$\mathbf{S}_x [+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_S, \Omega_S$$

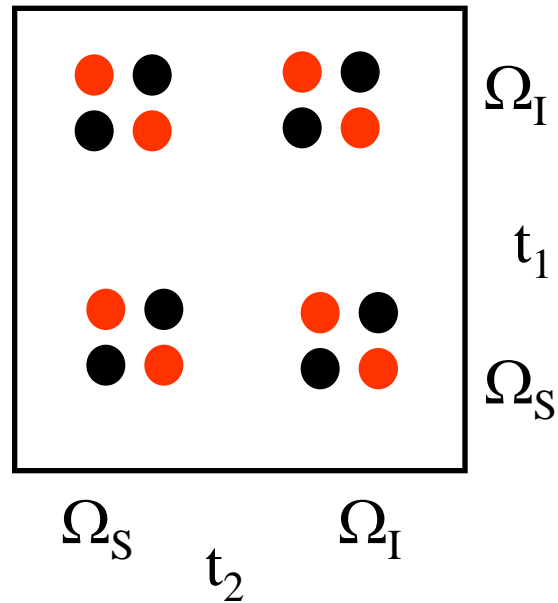
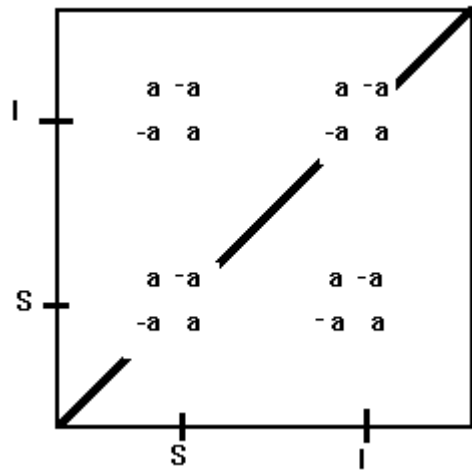
$$\mathbf{S}_x [+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_I, \Omega_S$$

so the diagonals as well as all off-diagonals have absorptive line shape



memo: line shapes
 positive absorptive (+a),
 negative absorptive (-a),
 positive dispersive (+d),
 negative dispersive (-d),





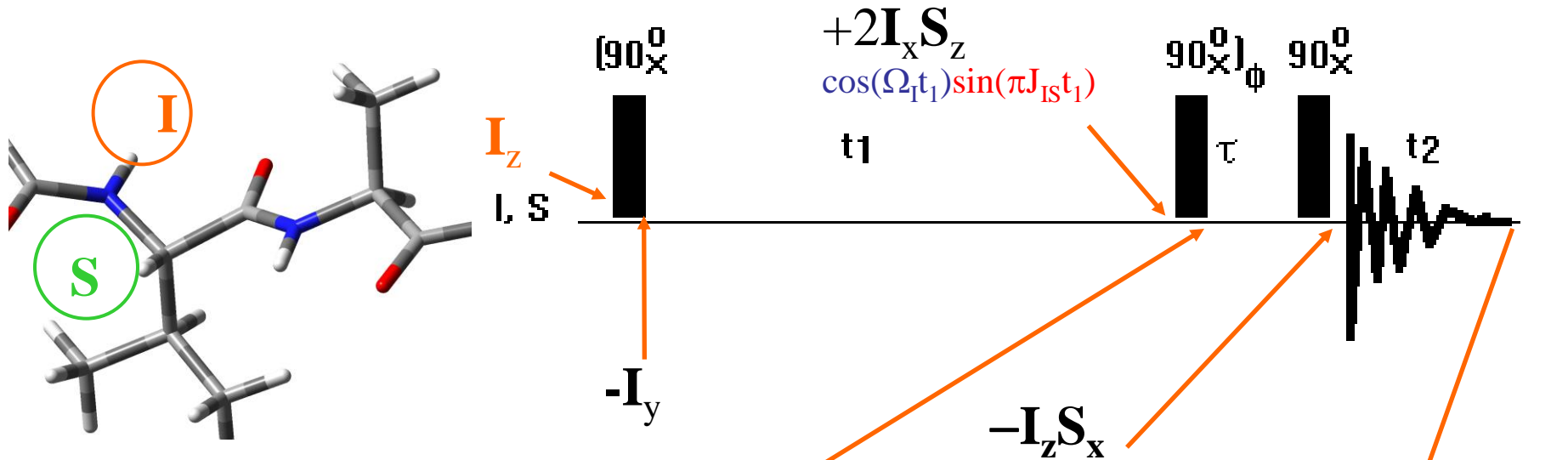
Comment:

The intensity of the DQF-COSY peaks are half of the appropriate COSY peaks ($1/8 \rightarrow 1/4$)

Phase cycled DQF-COSY is 4 time longer than the appropriate COSY.

Note: If selection is made not by phase-cycling but by "gradient", then the time requirement of the two experiments are identical.

Summary: ^1H - ^1H DQF-COSY the raise of an off-diagonal peak

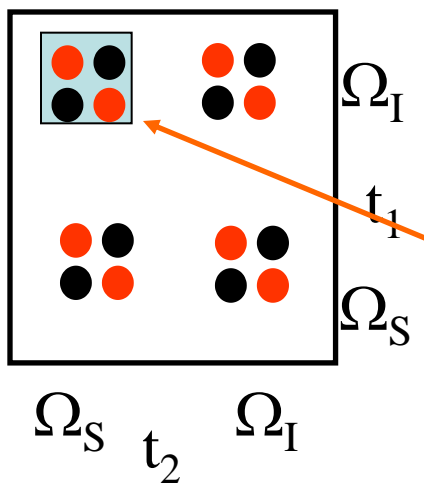


A DQ tagból a zQ tag kiemelését követően:

$$-2\mathbf{S}_x\mathbf{I}_y \Rightarrow -1/2 [2\mathbf{I}_x\mathbf{S}_y + 2\mathbf{I}_y\mathbf{S}_x]$$

$$-1/2\mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$-1/2\mathbf{S}_x \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{IS} t_2)$$



$$+1/8\mathbf{S}_x [+\sin\{(\Omega_I + \pi J_{IS})t_1\} - \sin\{(\Omega_I - \pi J_{IS})t_1\}] [+\cos\{(\Omega_S - \pi J_{IS})t_2\} - \cos\{(\Omega_S + \pi J_{IS})t_2\}]$$

