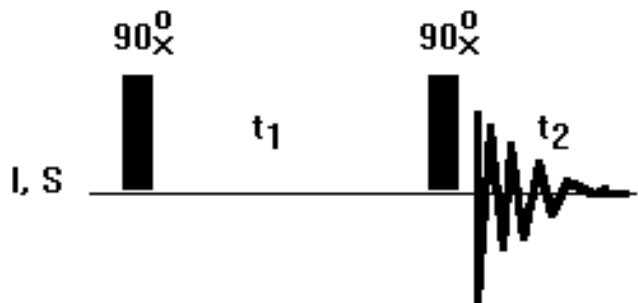
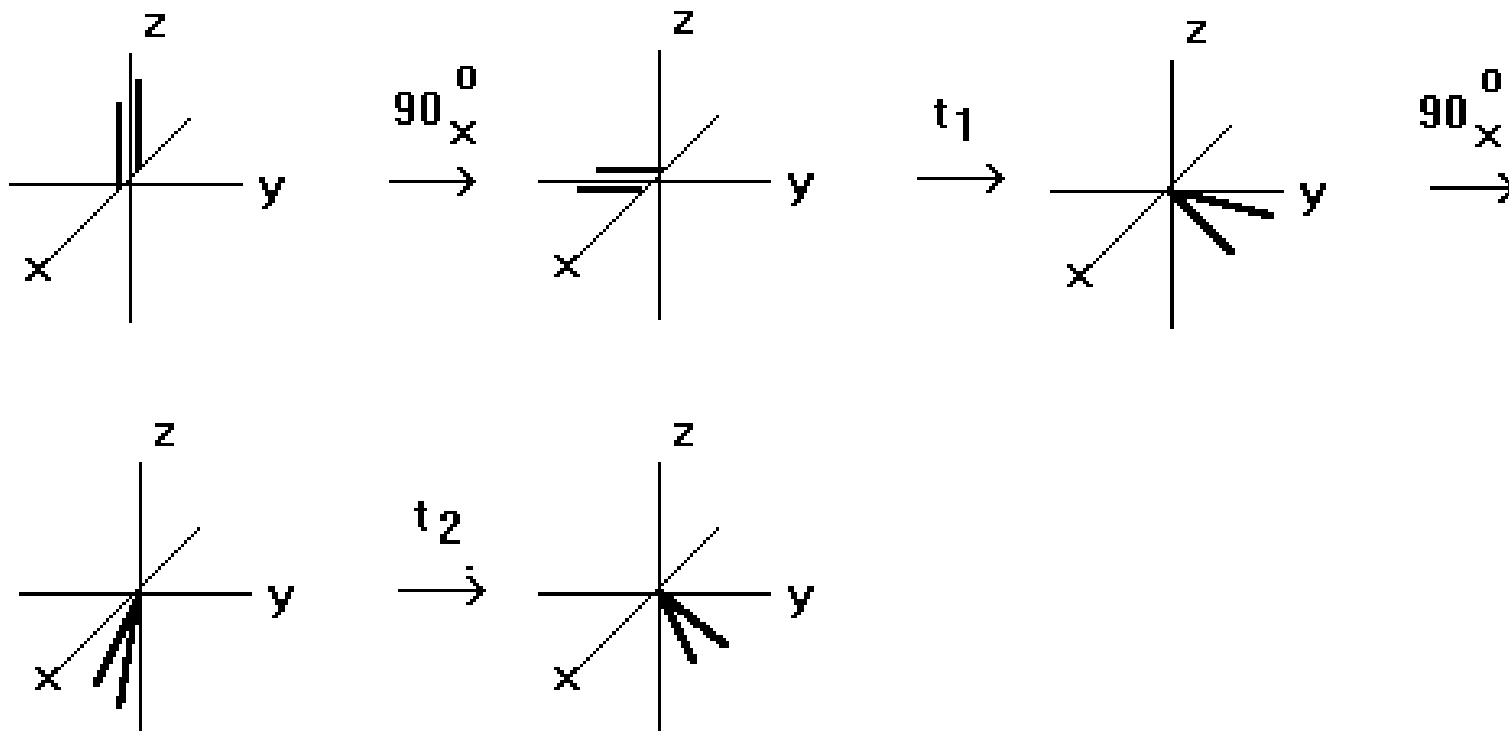


COSY = COrelated SpectroscopY

The pulse sequence: $90_x^\circ - t_1 - 90_x^\circ - t_2$



Consider: Ω_I , Ω_S and J_{IS}



$$\sigma[\text{eq.}]$$

$$\hat{H} = \pi/2 (\hat{I}_x + \check{S}_x)$$

$$\downarrow 90^\circ_x \quad \mathbf{I}_z \text{ and } \mathbf{S}_z$$

$$\sigma[0]$$

$$\hat{H} = \hat{I}_z(\Omega_I t_1) + \check{S}_z(\Omega_S t_1)$$

$$-\mathbf{I}_y \text{ and } -\mathbf{S}_y$$

$$\downarrow t_1$$

$$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi t_1)$$

$$-\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1)$$

and

$$-\mathbf{S}_y \cos(\Omega_S t_1) + \mathbf{S}_x \sin(\Omega_S t_1)$$

$$\downarrow$$

$$-\mathbf{I}_y \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1)$$

$$+ 2\mathbf{I}_x \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$+ \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1)$$

$$+ 2\mathbf{I}_y \mathbf{S}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

and

$$-\mathbf{S}_y \cos(\Omega_S t_1) \cos(\pi J_{IS} t_1)$$

$$+ 2\mathbf{S}_x \mathbf{I}_z \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1)$$

$$+ \mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1)$$

$$+ 2\mathbf{S}_y \mathbf{I}_z \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1)$$

$$\sigma[t_1]$$

$$\hat{H} = \pi/2 (\hat{I}_x + \hat{S}_x)$$

$$\downarrow 90^\circ_x \text{ memo.} = \cos(\pi/2) = 0, \sin(\pi/2) = 1$$

$$\begin{aligned} & -\mathbf{I}_z \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad -2\mathbf{I}_x \mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1) \\ & \quad +\mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad \quad -2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$

and

$$\begin{aligned} & -\mathbf{S}_z \cos(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad -2\mathbf{S}_x \mathbf{I}_y \cos(\Omega_S t_1) \sin(\pi J_{IS} t_1) \\ & \quad +\mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad \quad -2\mathbf{S}_z \mathbf{I}_y \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$

$$\sigma[t_1, 0]$$

memo. = Both \mathbf{I}_z and \mathbf{S}_z (z magnetization) as well as $\mathbf{I}_x \mathbf{S}_y$ and $\mathbf{S}_x \mathbf{I}_y$ (zero-double quant. coherences) will not result in observable signal, thus these 4 terms are ignored.

Therefore, the following 4 terms remain only :

$$\begin{aligned} & +\mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) \\ & \quad -2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$

and

$$\begin{aligned} & +\mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1) \\ & \quad -2\mathbf{S}_z \mathbf{I}_y \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1) \end{aligned}$$

$$\sigma[t_1, 0]$$

$$\hat{H} = \hat{I}_z(\Omega_I t_2) + \check{S}_z(\Omega_S t_2) \text{ and } 2\hat{I}_z\check{S}_z(\mathbf{J}_{IS}\pi t_2) \downarrow t_2$$

the \mathbf{I}_x term during ACQ \Rightarrow

$$\begin{aligned} & +\mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_I t_2) \cos(\pi \mathbf{J}_{IS} t_2) \quad \leftarrow \\ & +2\mathbf{I}_y \mathbf{S}_z \sin(\Omega_I t_1) \cos(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_I t_2) \sin(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{I}_y \sin(\Omega_I t_1) \cos(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_I t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & -2\mathbf{I}_x \mathbf{S}_z \sin(\Omega_I t_1) \cos(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi \mathbf{J}_{IS} t_2) \end{aligned}$$

the \mathbf{S}_x term during ACQ \Rightarrow

$$\begin{aligned} & +\mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi \mathbf{J}_{IS} t_2) \quad \leftarrow \\ & +2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_S t_1) \cos(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_S t_2) \sin(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{S}_y \sin(\Omega_S t_1) \cos(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_S t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & -2\mathbf{I}_z \mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi \mathbf{J}_{IS} t_2) \end{aligned}$$

the $-2\mathbf{I}_y \mathbf{S}_z$ term during ACQ \Rightarrow

$$\begin{aligned} & -2\mathbf{I}_y \mathbf{S}_z \sin(\Omega_S t_1) \sin(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_I t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{I}_x \sin(\Omega_S t_1) \sin(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_I t_2) \sin(\pi \mathbf{J}_{IS} t_2) \quad \leftarrow \\ & +2\mathbf{I}_x \mathbf{S}_z \sin(\Omega_S t_1) \sin(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_I t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{I}_y \sin(\Omega_S t_1) \sin(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_I t_2) \sin(\pi \mathbf{J}_{IS} t_2) \end{aligned}$$

the $-2\mathbf{I}_z \mathbf{S}_y$ term during ACQ \Rightarrow

$$\begin{aligned} & -2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi \mathbf{J}_{IS} t_1) \cos(\Omega_S t_2) \sin(\pi \mathbf{J}_{IS} t_2) \quad \leftarrow \\ & +2\mathbf{I}_z \mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_S t_2) \cos(\pi \mathbf{J}_{IS} t_2) \\ & +\mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi \mathbf{J}_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi \mathbf{J}_{IS} t_2) \end{aligned}$$

memo 1: if receiver is on x axis
the following four x terms remain:

$$\begin{aligned}
 & +\mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) \cos(\Omega_I t_2) \cos(\pi J_{IS} t_2) \\
 & +\mathbf{S}_x \sin(\Omega_S t_1) \cos(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{IS} t_2) \\
 & +\mathbf{I}_x \sin(\Omega_S t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_I t_2) \sin(\pi J_{IS} t_2) \\
 & +\mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \sin(\pi J_{IS} t_2)
 \end{aligned}$$

memo 2:

$$\begin{aligned}
 \sin(A)\cos(B) &= 1/2[\sin(A+B)+\sin(A-B)] \\
 \cos(A)\sin(B) &= 1/2[\sin(A+B)-\sin(A-B)] \\
 \sin(A)\sin(B) &= 1/2[\cos(A-B)-\cos(A+B)] \\
 \cos(A)\cos(B) &= 1/2[\cos(A+B)+\cos(A-B)]
 \end{aligned}$$

therefore

$$\begin{aligned}
 & +1/4\mathbf{I}_x[+\sin\{(\Omega_I+\pi J_{IS})t_1\}+\sin\{(\Omega_I-\pi J_{IS})t_1\}][+\cos\{(\Omega_I+\pi J_{IS})t_2\}+\cos\{(\Omega_I-\pi J_{IS})t_2\}] \\
 & +1/4\mathbf{S}_x[+\sin\{(\Omega_S+\pi J_{IS})t_1\}+\sin\{(\Omega_S-\pi J_{IS})t_1\}][+\cos\{(\Omega_S+\pi J_{IS})t_2\}+\cos\{(\Omega_S-\pi J_{IS})t_2\}] \\
 & +1/4\mathbf{I}_x[+\cos\{(\Omega_S-\pi J_{IS})t_1\}-\cos\{(\Omega_S+\pi J_{IS})t_1\}][+\sin\{(\Omega_I+\pi J_{IS})t_2\}-\sin\{(\Omega_I-\pi J_{IS})t_2\}] \\
 & +1/4\mathbf{S}_x[+\cos\{(\Omega_I-\pi J_{IS})t_1\}-\cos\{(\Omega_I+\pi J_{IS})t_1\}][+\sin\{(\Omega_S+\pi J_{IS})t_2\}-\sin\{(\Omega_S-\pi J_{IS})t_2\}]
 \end{aligned}$$

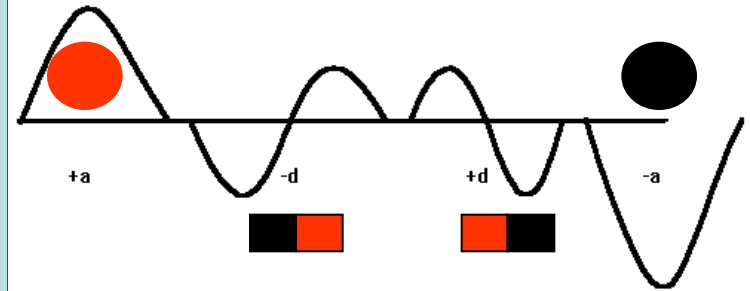
the following terms can be found

$$\begin{aligned}
 & \mathbf{I}_x [+ .. + .. + .. + ..] \text{ at } \Omega_I, \Omega_I \\
 & \mathbf{I}_x [+ .. - .. + .. - ..] \text{ at } \Omega_S, \Omega_I \\
 & \mathbf{S}_x [+ .. + .. + .. + ..] \text{ at } \Omega_S, \Omega_S \\
 & \mathbf{S}_x [+ .. - .. + .. - ..] \text{ at } \Omega_I, \Omega_S
 \end{aligned}$$

$$\begin{aligned}
& +1/4\mathbf{I}_x[+\sin\{(\Omega_I+\pi\mathbf{J}_{IS})t_1\}+\sin\{(\Omega_I-\pi\mathbf{J}_{IS})t_1\}] [+ \cos\{(\Omega_I+\pi\mathbf{J}_{IS})t_2\}+\cos\{(\Omega_I-\pi\mathbf{J}_{IS})t_2\}] \\
& +1/4\mathbf{I}_x[+\cos\{(\Omega_S-\pi\mathbf{J}_{IS})t_1\}-\cos\{(\Omega_S+\pi\mathbf{J}_{IS})t_1\}][+\sin\{(\Omega_I+\pi\mathbf{J}_{IS})t_2\}-\sin\{(\Omega_I-\pi\mathbf{J}_{IS})t_2\}] \\
& +1/4\mathbf{S}_x[+\sin\{(\Omega_S+\pi\mathbf{J}_{IS})t_1\}+\sin\{(\Omega_S-\pi\mathbf{J}_{IS})t_1\}][+\cos\{(\Omega_S+\pi\mathbf{J}_{IS})t_2\}+\cos\{(\Omega_S-\pi\mathbf{J}_{IS})t_2\}] \\
& +1/4\mathbf{S}_x[+\cos\{(\Omega_I-\pi\mathbf{J}_{IS})t_1\}-\cos\{(\Omega_I+\pi\mathbf{J}_{IS})t_1\}][+\sin\{(\Omega_S+\pi\mathbf{J}_{IS})t_2\}-\sin\{(\Omega_S-\pi\mathbf{J}_{IS})t_2\}]
\end{aligned}$$

memo: line shapes

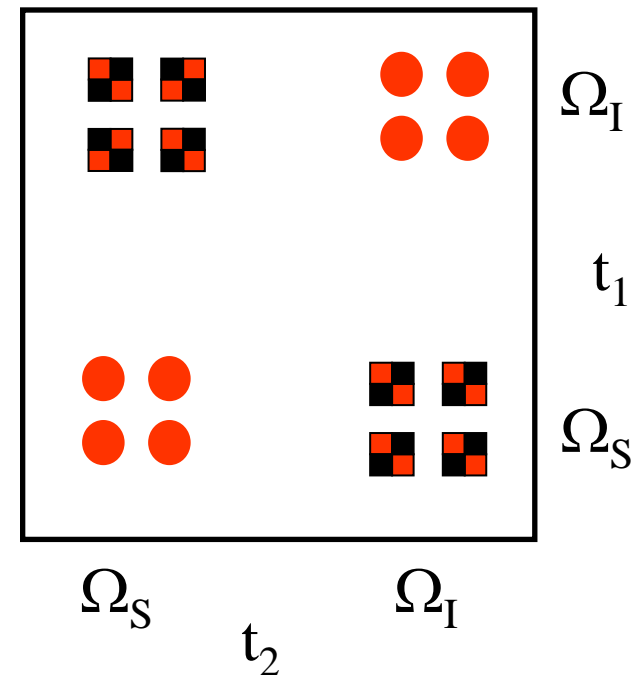
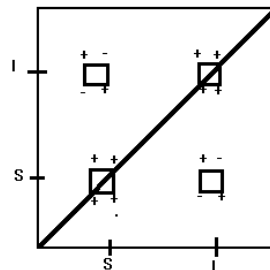
*positive absorptive (+a), negative absorptive (-a),
positive dispersive (+d), negative dispersive (-d),*



if one sets the phase that **sin is **absorptive** (a) in t_1 and **cos** is **absorptive** (a) in t_2 then:**

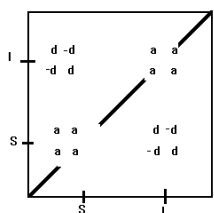
$$\begin{aligned}
\mathbf{I}_x [+a \dots +a \dots +a \dots +a \dots] & \text{ at } \Omega_I, \Omega_I \\
\mathbf{I}_x [+d \dots -d \dots +d \dots -d \dots] & \text{ at } \Omega_S, \Omega_I \\
\mathbf{S}_x [+a \dots +a \dots +a \dots +a \dots] & \text{ at } \Omega_S, \Omega_S \\
\mathbf{S}_x [+d \dots -d \dots +d \dots -d \dots] & \text{ at } \Omega_I, \Omega_S
\end{aligned}$$

so the diagonal peaks
have absorptive and all
the off-diagonals have
dispersive line shape.

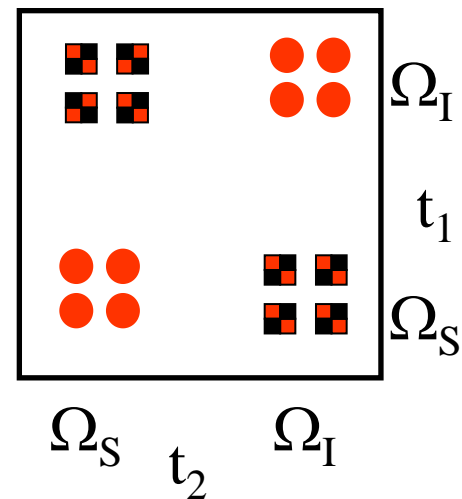


$$\begin{aligned}
&+1/4\mathbf{I}_x[+\sin\{(\Omega_I+\pi\mathbf{J}_{IS})t_1\}+\sin\{(\Omega_I-\pi\mathbf{J}_{IS})t_1\}] [+ \cos\{(\Omega_I+\pi\mathbf{J}_{IS})t_2\}+\cos\{(\Omega_I-\pi\mathbf{J}_{IS})t_2\}] \\
&+1/4\mathbf{I}_x[+\cos\{(\Omega_S-\pi\mathbf{J}_{IS})t_1\}-\cos\{(\Omega_S+\pi\mathbf{J}_{IS})t_1\}][+\sin\{(\Omega_I+\pi\mathbf{J}_{IS})t_2\}-\sin\{(\Omega_I-\pi\mathbf{J}_{IS})t_2\}] \\
&+1/4\mathbf{S}_x[+\sin\{(\Omega_S+\pi\mathbf{J}_{IS})t_1\}+\sin\{(\Omega_S-\pi\mathbf{J}_{IS})t_1\}][+\cos\{(\Omega_S+\pi\mathbf{J}_{IS})t_2\}+\cos\{(\Omega_S-\pi\mathbf{J}_{IS})t_2\}] \\
&+1/4\mathbf{S}_x[+\cos\{(\Omega_I-\pi\mathbf{J}_{IS})t_1\}-\cos\{(\Omega_I+\pi\mathbf{J}_{IS})t_1\}][+\sin\{(\Omega_S+\pi\mathbf{J}_{IS})t_2\}-\sin\{(\Omega_S-\pi\mathbf{J}_{IS})t_2\}]
\end{aligned}$$

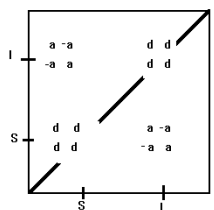
if one sets the phase that \sin is absorptive (a) in t_1 and \cos is absorptive (a) in t_2 :



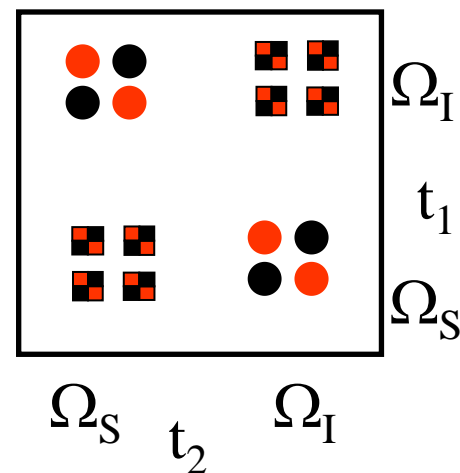
$$\begin{aligned}
\mathbf{I}_x &[+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_I, \Omega_I \\
\mathbf{I}_x &[+d \dots -d \dots +d \dots -d \dots] \text{ at } \Omega_S, \Omega_I \\
\mathbf{S}_x &[+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_S, \Omega_S \\
\mathbf{S}_x &[+d \dots -d \dots +d \dots -d \dots] \text{ at } \Omega_I, \Omega_S
\end{aligned}$$



if one sets the phase that \cos is absorptive (a) in t_1 and \sin is absorptive (a) in t_2



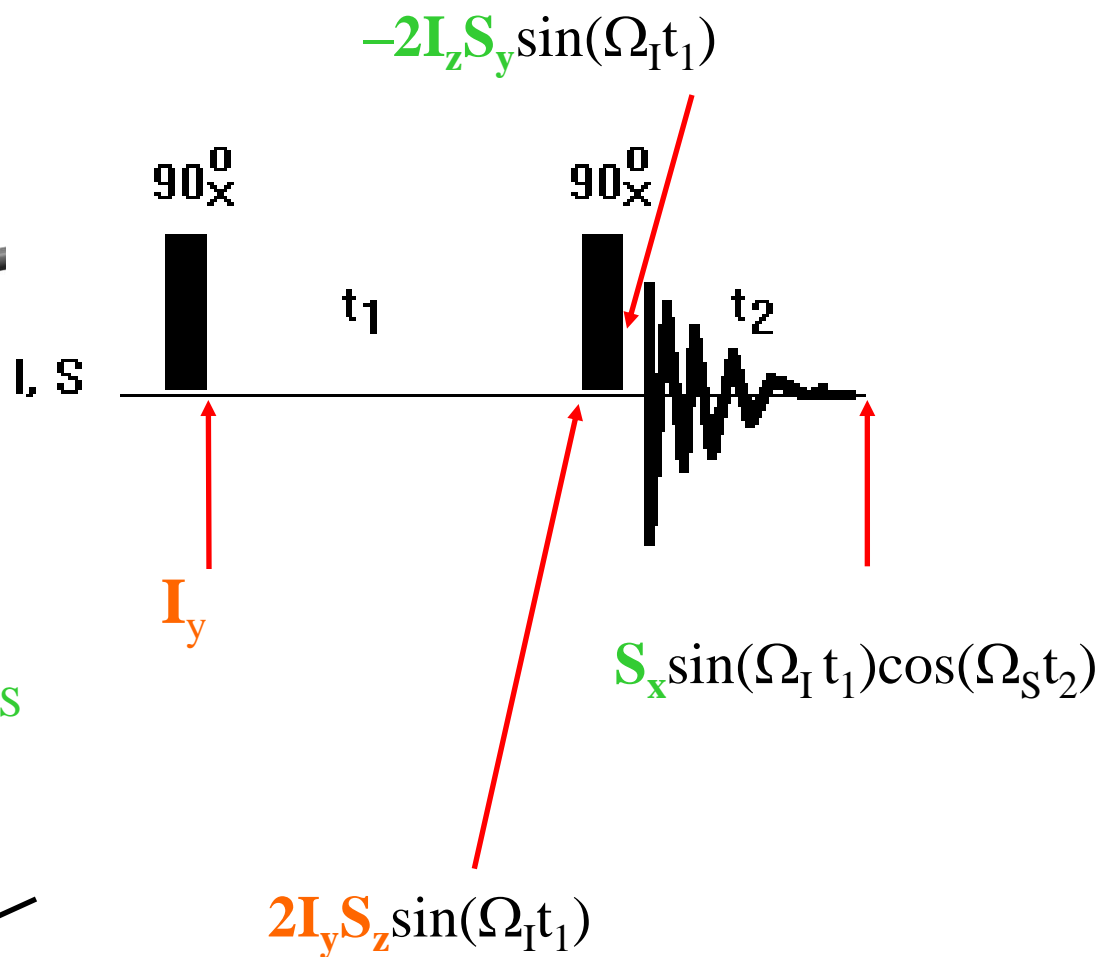
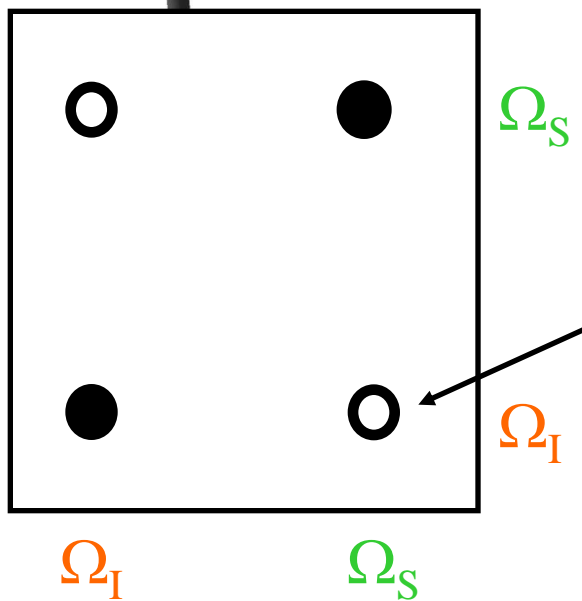
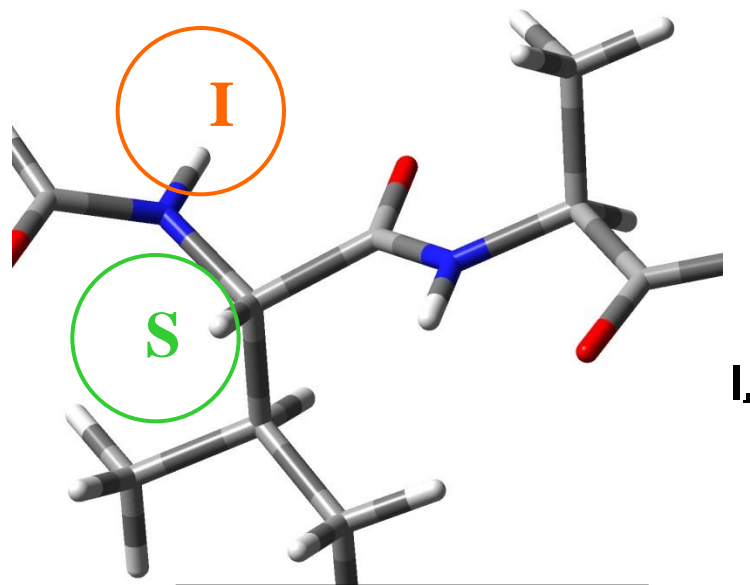
$$\begin{aligned}
\mathbf{I}_x &[+d \dots +d \dots +d \dots +d \dots] \text{ at } \Omega_I, \Omega_I \\
\mathbf{I}_x &[+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_S, \Omega_I \\
\mathbf{S}_x &[+d \dots +d \dots +d \dots +d \dots] \text{ at } \Omega_S, \Omega_S \\
\mathbf{S}_x &[+a \dots -a \dots +a \dots -a \dots] \text{ at } \Omega_I, \Omega_S
\end{aligned}$$

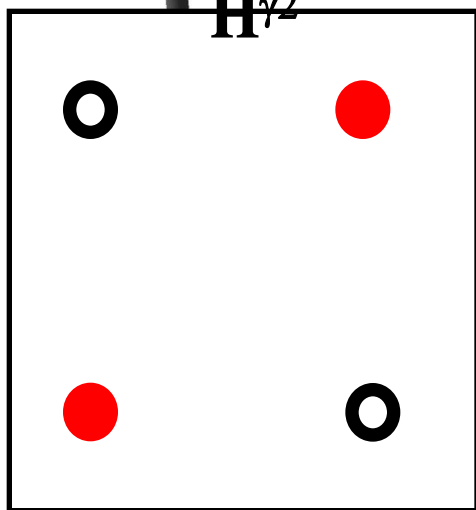
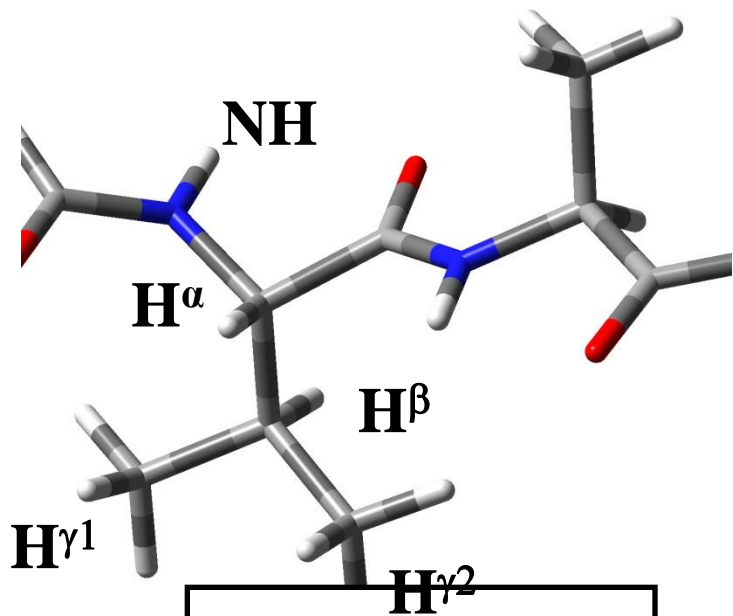


so now the diagonals have dispersive and all off-diagonals have absorptive line shape

Comment : even if the diagonals have dispersive and the off-diagonal peaks have absorptive line-shape, the off-diagonal peaks near the diagonal will have poor resolution because of the poor base line.

Összefoglaló I: ^1H - ^1H COSY (homonukleáris korrelációs spektrum)



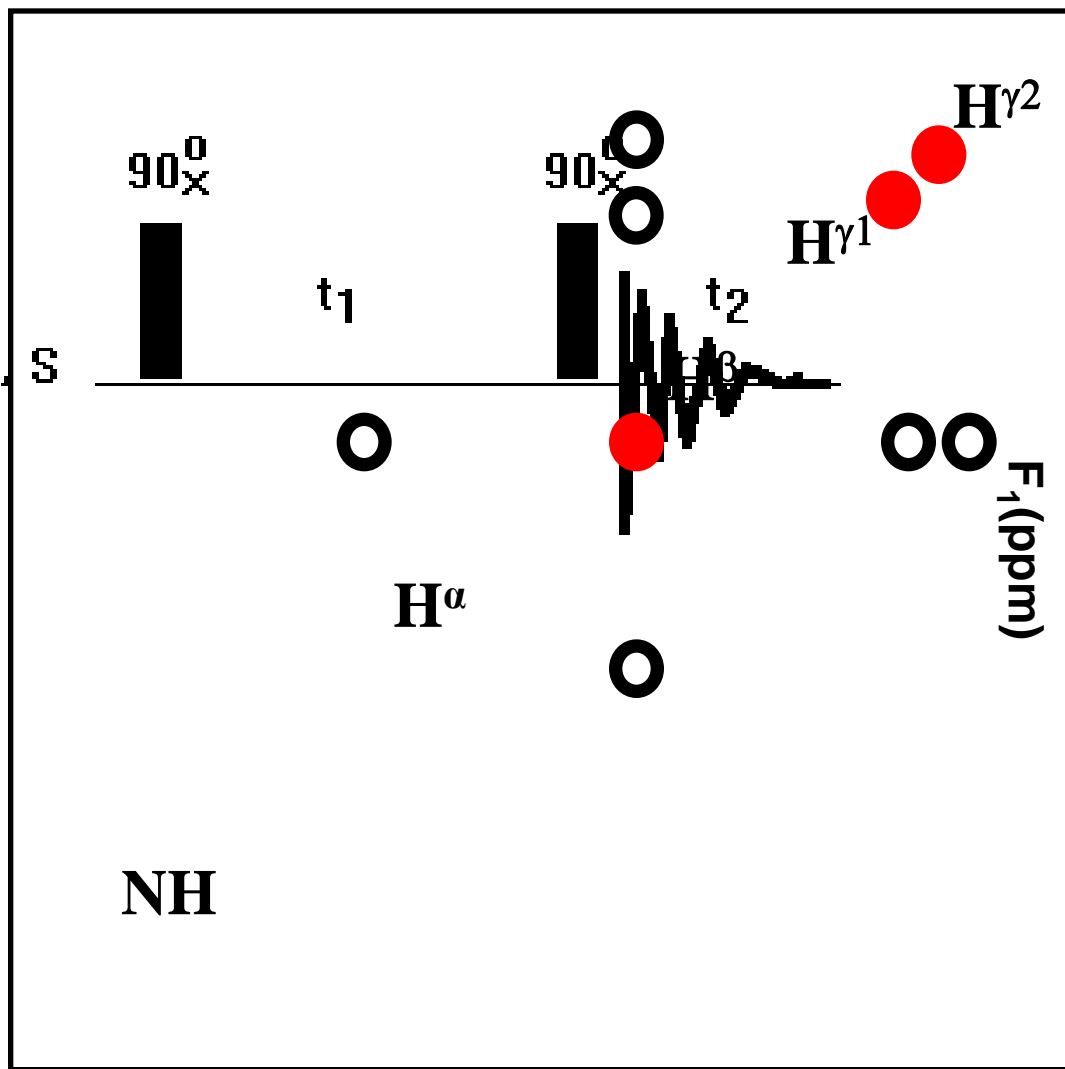


Ω_I

Ω_S

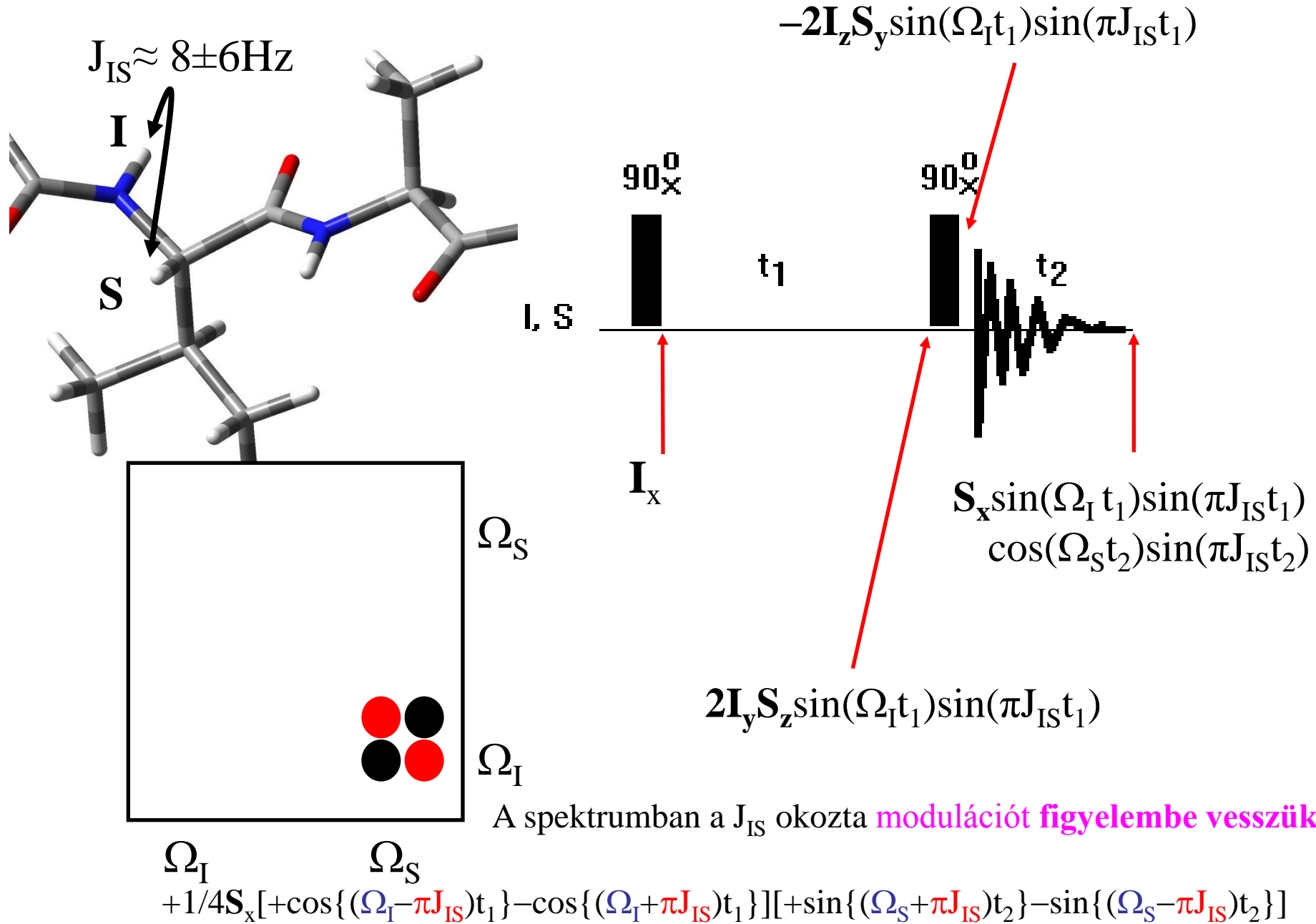
Ω_S

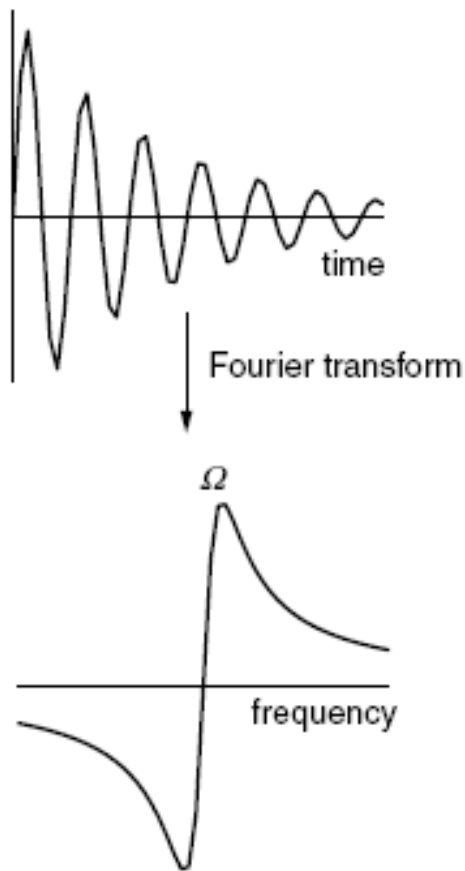
Ω_I



A spektrumban a J_{IS} okozta modulációtól eltekintünk

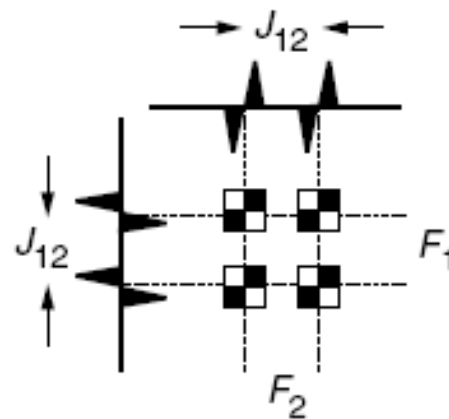
Összefoglaló II: ^1H - ^1H COSY (homonukleáris korrelációs spektrum)



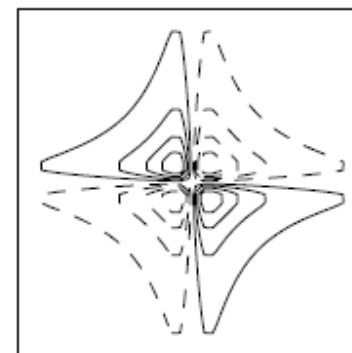
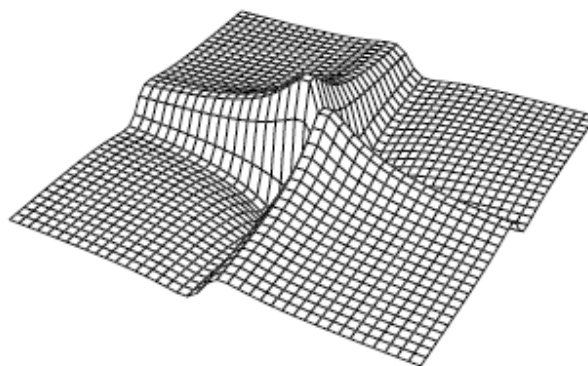


The Fourier transform of a decaying sine function $\sin\Omega t \exp(-t/T_2)$ is a dispersion mode Lorentzian centred at frequency Ω .

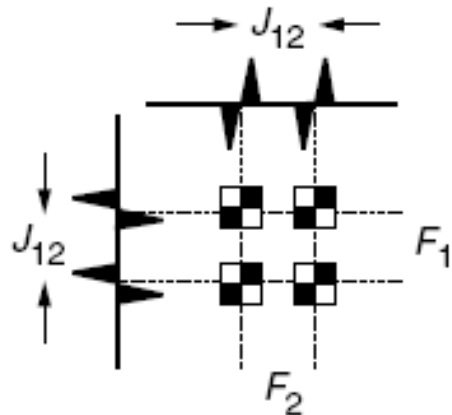
frequency ω .



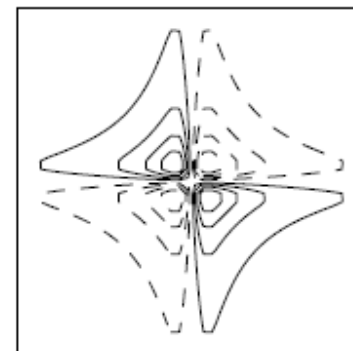
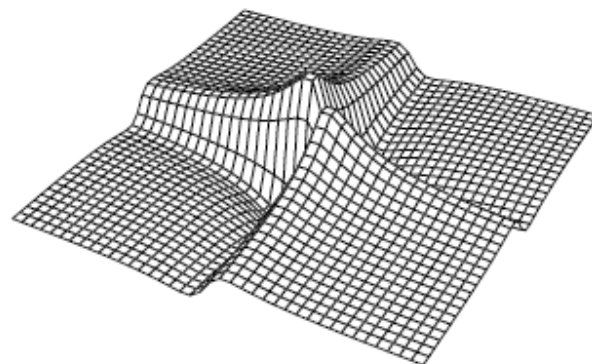
Schematic view of the diagonal peak from a COSY spectrum. The squares are supposed to indicate the two-dimensional double dispersion lineshape illustrated below



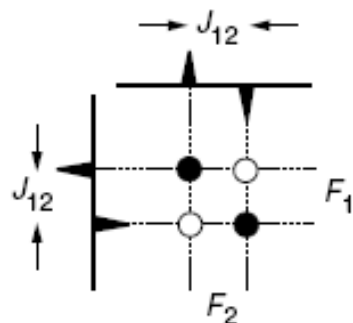
The double dispersion lineshape seen in pseudo 3D and as a contour plot; negative contours are indicated by dashed lines.



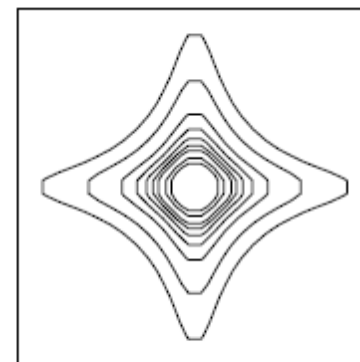
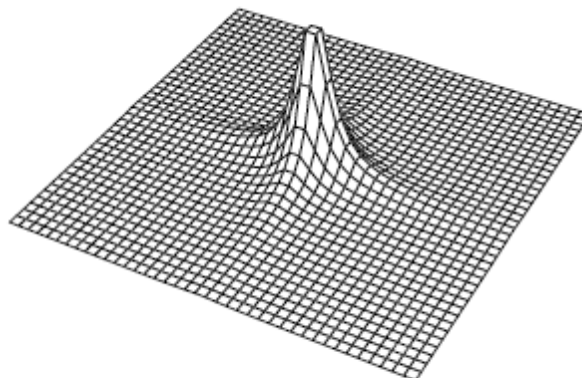
Schematic view of the diagonal peak from a COSY spectrum. The squares are supposed to indicate the two-dimensional double dispersion lineshape illustrated below



The double dispersion lineshape seen in pseudo 3D and as a contour plot; negative contours are indicated by dashed lines.



Schematic view of the cross-peak multiplet from a COSY spectrum. The circles are supposed to indicate the two-dimensional double absorption lineshape illustrated below; filled circles represent positive intensity, open represent negative intensity.



The double absorption lineshape seen in pseudo 3D and as a contour plot.

