

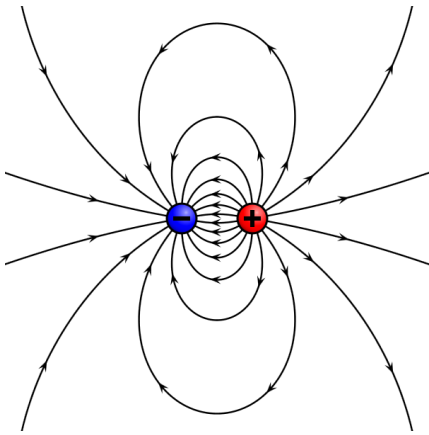
Preamble:

B: a **mágneses indukció** (mágneses fluxussűrűség), a mágneses mező (mágneses erőtér) a mozgó elektromos töltés, vagy az elektromos mező változásának következménye.

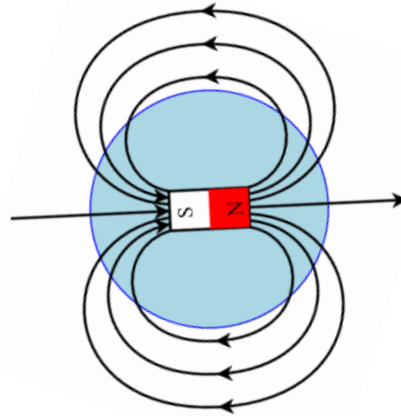
SI egysége a tesla (T)

- $1 \text{ T} = 1 \text{ Vs/m}^2 = 1 \text{ kg} \cdot \text{s}^{-2} \cdot \text{A}^{-1}$ ($1 \text{ T} = 10^4$ gauss (G) (CGS egysége)),
- a Föld mágneses mezeje 50° szélességen $20 \mu\text{T}$ (0,2G), 0° szélességen $31 \mu\text{T}$,
- patkómágnes (10G), napfolt (0,1- 10 T), neutroncsillag ($10^6 \text{ T} - 10^8 \text{ T}$),

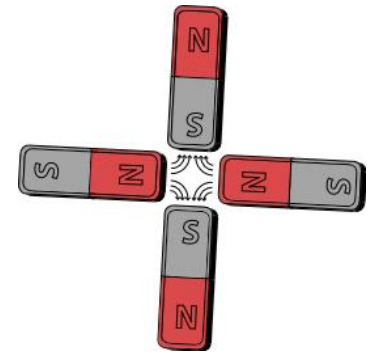
H: mágneses térerősség (SI egysége az A/m) $B = \mu\mu_0 H$



The magnetic pole model: two opposing poles, North (+) and South (-), separated by a distance d produce an **H**-field (lines).



A sketch of Earth's magnetic field representing the source of the field as a magnet. The geographic north pole of Earth is near the top of the diagram, the south pole near the bottom. The south pole of that magnet is deep in Earth's interior below Earth's North Magnetic Pole



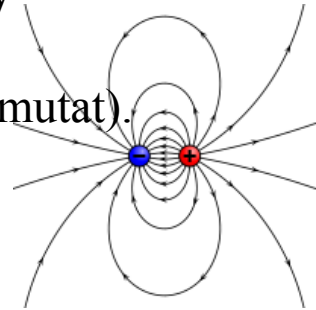
Schematic quadrupole magnet ("four-pole") magnetic field. There are four steel pole tips, two opposing magnetic north poles and two opposing magnetic south poles

Egy mágnes **mágneses momentuma**

- az az erő, amellyel hatást gyakorol az áramra,
- az a nyomaték, amivel a mágneses mező hat rá.

Mágneses momentuma van a Földnek, egy rúd-mágnesnek, egy molekulának, egy elektronnak, stb..

Vektormennyiség: van nagysága és iránya (a mágnes déli sarkától az északi felé mutat).



A **mágneses momentum** ($\text{Am}^2 = \text{J/T}$): két ellenkező **pólust** (**töltést**) véges tér választ el egymástól. (**elektrosztatikai analógia**) (**memo**: az elektrosztatikával analóg módon a mágneses momentum forrását itt is pólusok alkotják.)

Pl. tekintsünk egy rúd-mágnest, amelynek két ellentétes mágneses pólusa van egyenlő nagyságrendben. Mindegyik pólus a mágneses erő forrása, amely a távolsággal gyengül. (A mágneses pólusok mindig párban vannak, s ezért kiegyenlítik egymást.) Ez a kiegyenlítő erő annál nagyobb, minél közelebb vannak a pólusok egymáshoz, azaz, minél rövidebb a rúd.

A mágnesrúd által keltett mágneses erő a tér egy pontján két tényezőtől függ:

- a pólusai erejétől (p),
- az őket elkülönítő vektortól (\mathbf{l}).

Így a momentum: **$\boldsymbol{\mu} = \mathbf{p} \mathbf{l}$**

A mágneses dipólusoknak van impulzusnyomatékuk.

initial comment:

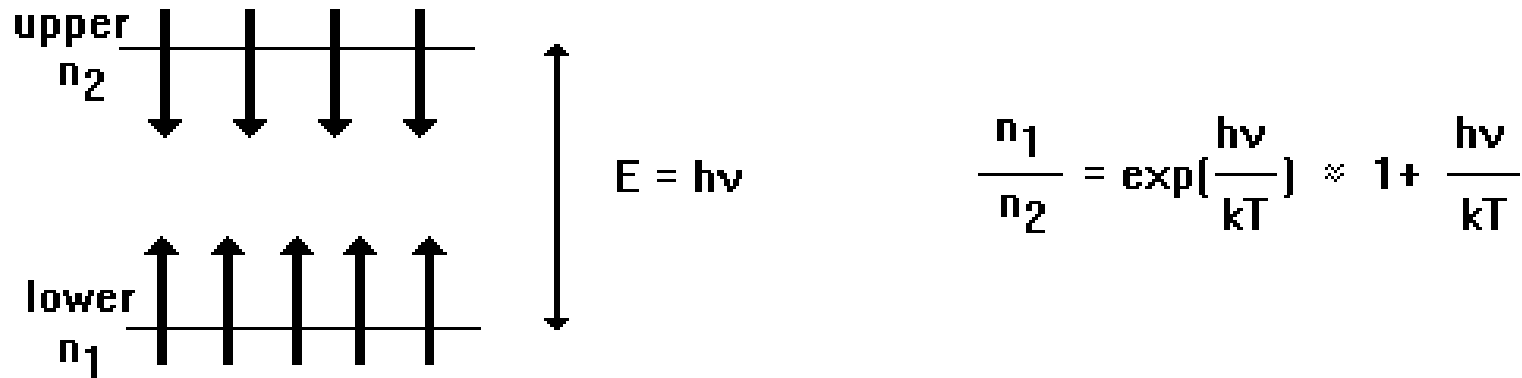
BLOCH EQUATIONS

the phenomenological description of the **macroscopic event**,
valid only for **isolated, non-coupled** nuclei.

aim: description of the magnetic resonance phenomenon.

initial status:

Boltzman distribution of the lower and the upper spin states at thermal equilibrium:



conclusion :

- this excess of nuclei generate a **bulk magnetic moment** (M_o)
- $M_o \parallel B_o$ (equilibrium Z magnetization [M_o] is parallel to the external field [B_o]).
- NMR is **insensitive**: approx. a unit **excess** for an ensemble of **1E+9** spins³
(at normal temp.)!

Equation of motion: $dM/dt = \gamma[M \otimes B]$

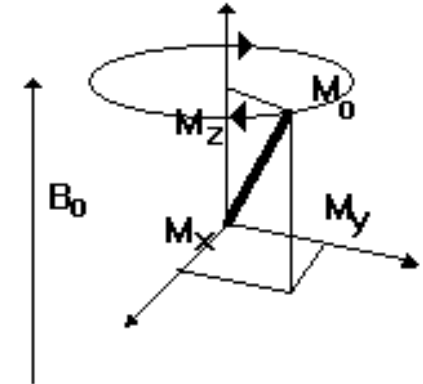
describing the behavior of M in the presence of B

memo 1: vektor vektoriális szorzata:

$a := (2, -1, 3)$ és $b := (1, 0, 7)$

$$a \otimes b = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ 1 & 0 & 7 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 0 & 7 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} \mathbf{k}$$

$$(-7\mathbf{i}) - (11\mathbf{j}) + (1\mathbf{k}) = \mathbf{(-7, -11, 1)}$$



memo 2: vektor vektoriális szorzata:

$M := (M_x, M_y, M_z)$ és $B := (B_x, B_y, B_z)$

$$M \otimes B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} M_y & M_z \\ B_y & B_z \end{vmatrix} \mathbf{i} - \begin{vmatrix} M_x & M_z \\ B_x & B_z \end{vmatrix} \mathbf{j} + \begin{vmatrix} M_x & M_y \\ B_x & B_y \end{vmatrix} \mathbf{k}$$

$$(\mathbf{M}_y \mathbf{B}_z - \mathbf{B}_y \mathbf{M}_z) \mathbf{i} - (\mathbf{M}_x \mathbf{B}_z - \mathbf{B}_x \mathbf{M}_z) \mathbf{j} + (\mathbf{M}_x \mathbf{B}_y - \mathbf{B}_x \mathbf{M}_y) \mathbf{k}$$

Conclusion: $dM/dt = \gamma[M \otimes B]$

$$dM_x/dt = \gamma [M_y B_z - M_z B_y]$$

$$dM_y/dt = \gamma [M_z B_x - M_x B_z]$$

$$dM_z/dt = \gamma [M_x B_y - M_y B_x]$$

A: in the absence of exciting field (no B_1), **B** is $B = (0,0,B_0)$.

$$dM_x/dt = \gamma [M_y B_z - M_z B_y]$$

$$dM_y/dt = \gamma [M_z B_x - M_x B_z]$$

$$dM_z/dt = \gamma [M_x B_y - M_y B_x]$$

as B_x and $B_y = 0$ and $B_z = B_0$

$$dM_x/dt = \gamma B_0 M_y$$

M_y rotates about B_0 (Larmor precession)

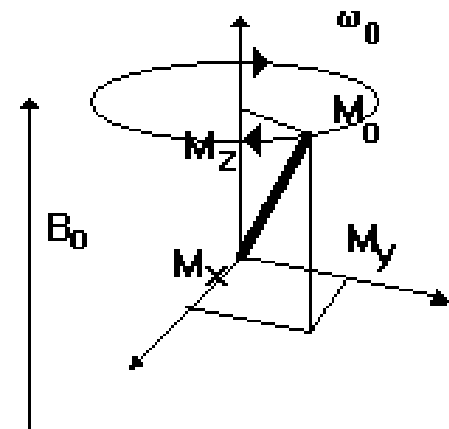
$$dM_y/dt = -\gamma B_0 M_x$$

M_x rotates about B_0 (Larmor precession)

$$dM_z/dt = 0$$

M_z is constant

Memo: M vektor B körül precesszál,
de mivel $B = B_0$ ebben az esetben, és B_0
párhuzamos 'z'-vel ezért ténylegesen M a z- tengely
körül precesszál ω_0 szögsebességgel



B: in the presence of an exciting field ($2B_1$),
with components B_{1x} and B_{1y} and thus,

$$\mathbf{B} = (\mathbf{B}_x, \mathbf{B}_y, \mathbf{B}_z) = (\mathbf{B}_{1x}, \mathbf{B}_{1y}, \mathbf{B}_0)$$

$$dM_x/dt = \gamma [M_y B_z - M_z B_y]$$

$$dM_x/dt = \gamma [M_y B_0 - M_z B_{1y}]$$

$$dM_y/dt = \gamma [M_z B_x - M_x B_z]$$

$$dM_y/dt = \gamma [M_z B_{1x} - M_x B_0]$$

$$dM_z/dt = \gamma [M_x B_y - M_y B_x]$$

$$dM_z/dt = \gamma [M_x B_{1y} - M_y B_{1x}]$$

$$dM_x/dt = \gamma B_0 M_y - \gamma B_{1y} M_z$$

M_y rotates about B_0 and M_z rotates about B_1

$$dM_y/dt = -\gamma B_0 M_x \pm \gamma B_{1x} M_z$$

M_x rotates about B_0 and M_z rotates about B_1

$$dM_z/dt = \pm \gamma B_{1y} M_x \mp \gamma B_{1x} M_y$$

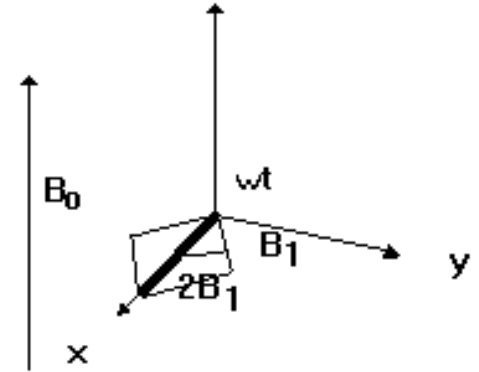
M_x rotates about B_1 and M_y rotates about B_1

$$B_{1x} = B_1 \cos(\omega t) \quad \text{and} \quad B_{1y} = -B_1 \sin(\omega t)$$

$$dM_x/dt = \gamma B_0 M_y + \gamma B_1 M_z \sin(\omega t)$$

$$dM_y/dt = -\gamma B_0 M_x \pm \gamma B_1 M_z \cos(\omega t)$$

$$dM_z/dt = \mp \gamma B_1 M_x \sin(\omega t) \mp \gamma B_1 M_y \cos(\omega t)$$



$$dM_x/dt = \gamma [B_0 M_y + B_1 M_z \sin(\omega t)]$$

$$dM_y/dt = -\gamma [B_0 M_x \mp B_1 M_z \cos(\omega t)]$$

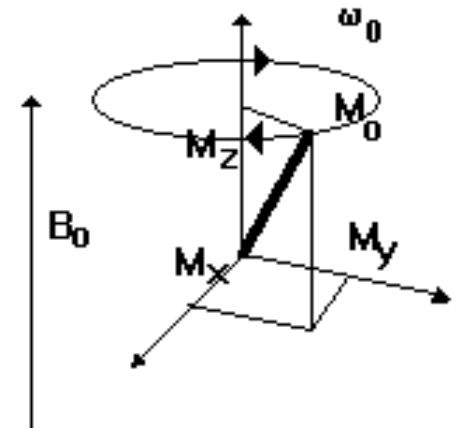
$$dM_z/dt = -\gamma [\pm B_1 M_x \sin(\omega t) \pm B_1 M_y \cos(\omega t)]$$

memo: hogy is precessál M ?, mely tengelyek körül precesszál M ?

$$\begin{aligned}
 dM_x/dt &= \gamma[B_0 M_y + B_1 M_z \sin(\omega t)] & M_y \text{ rotates about } B_0 \text{ and } M_z \text{ rotates about } B_1 \\
 dM_y/dt &= -\gamma[B_0 M_x - B_1 M_z \cos(\omega t)] & M_x \text{ rotates about } B_0 \text{ and } M_z \text{ rotates about } B_1 \\
 dM_z/dt &= -\gamma[\pm B_1 M_x \sin(\omega t) \pm B_1 M_y \cos(\omega t)] & M_x \text{ rotates about } B_1 \text{ and } M_y \text{ rotates about } B_1
 \end{aligned}$$

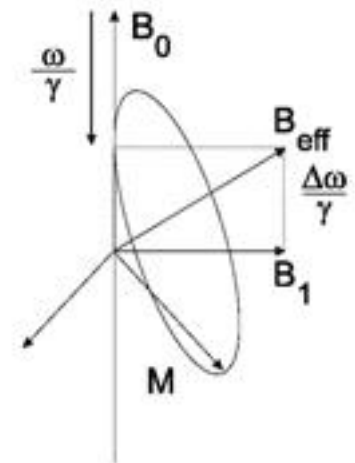
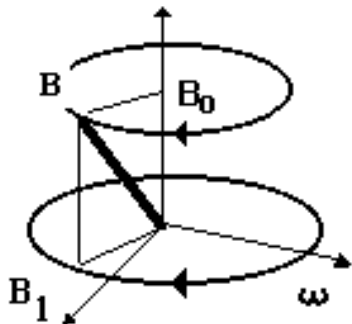
Ha $B_0 \gg B_1$ akkor lényegében $B \sim (0, 0, B_0)$
és azt a képet kapjuk mint **A**) esetben:

Memo: M vektor B körül precesszál,
ám $B \sim B_0$ s ezért M a $\sim z$ - tengely körül precesszál
közel ω_0 szögsebességgel



Ha $B_0 > B_1$ akkor már az x és az y komponensek $\neq 0$
 $B (B_{1x}, B_{1y}, B_0)$ és új helyzet áll elő:

Memo: M vektor B vagy B_{eff} körül precesszál,
ám B már nem párhuzamos a z - tengellyel!
Ezért M nem csak a $\sim z$ - tengely körül precesszál.
Hanem az ω_1 szögsebességgel forgó B - (B_{eff})-körüli!



C: Using \mathbf{B} as $(\mathbf{B}_{1x}, \mathbf{B}_{1y}, \mathbf{B}_0)$ and

introducing **two relaxation time constants:**

- longitudinal (z) or T_1
- transverse (x, y) or T_2

$$\begin{aligned} dM_x/dt &= \gamma[B_0 M_y + B_1 M_z \sin(\omega t)] \\ dM_y/dt &= -\gamma[B_0 M_x - B_1 M_z \cos(\omega t)] \\ dM_z/dt &= -\gamma[\pm B_1 M_x \sin(\omega t) \pm B_1 M_y \cos(\omega t)] \end{aligned}$$

$$\begin{aligned} dM_x/dt &= \gamma[B_0 M_y + B_1 M_z \sin(\omega t)] - M_x/T_2 \\ dM_y/dt &= -\gamma[B_0 M_x - B_1 M_z \cos(\omega t)] - M_y/T_2 \\ dM_z/dt &= -\gamma[\pm B_1 M_x \sin(\omega t) \pm B_1 M_y \cos(\omega t)] - (M_z - M_0)/T_1 \end{aligned}$$

**This is the Bloch equation in an external x, y, z coordinate
(in the laboratory frame).**

memo: ahogy láttuk ez egy igen komplex mozgás, lehet-e egyszerűsíteni?

pontosabban: lehet-e célszerűbb koordinátarendszert választanunk?

$$\Delta E_{\alpha \rightarrow \beta} = E_{\beta} - E_{\alpha} = +\frac{1}{2} \hbar \gamma B_0 - (-\frac{1}{2} \hbar \gamma B_0) = \hbar \gamma B_0$$

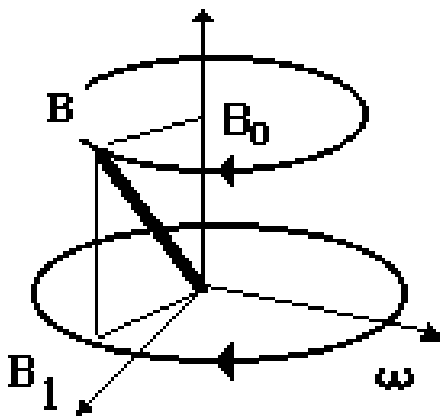
$$\hbar \nu_0 = (\hbar/2\pi) \gamma B_0$$

$$\nu_0 = \gamma B_0 / 2\pi$$

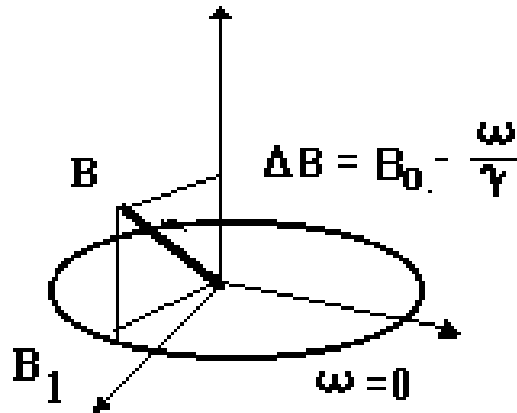
$$\omega_0 = \gamma B_0$$

$$\omega / \gamma = B$$

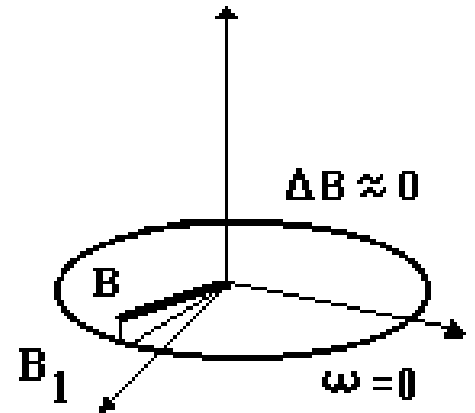
Tehát $\Delta B = (\omega_0 - \omega) / \gamma$



nyugvó referenciarendszer
(ahol B (B_0 és B_1 eredője) precesszál)



forgó referenciarendszer
(ahol B nem precesszál)



A B_0 nagysága relatív: attól függ hogy melyik koordinátarendszerből nézzük

He tehát a „lila” egyenletbe B_0 helyére ΔB -t írunk, akkor:

$$\begin{aligned} dM_x'/dt &= \gamma[\Delta B M_y' + B_1 M_z' \sin(\omega t)] - M_x'/T_2 \\ dM_y'/dt &= -\gamma[\Delta B M_x' + B_1 M_z' \cos(\omega t)] - M_y'/T_2 \\ dM_z'/dt &= -\gamma[-B_1 M_x' \sin(\omega t) - B_1 M_y' \cos(\omega t)] - (M_z' - M_0)/T_1 \end{aligned}$$

és mivel $\Delta B = (\omega_0 - \omega)/\gamma$

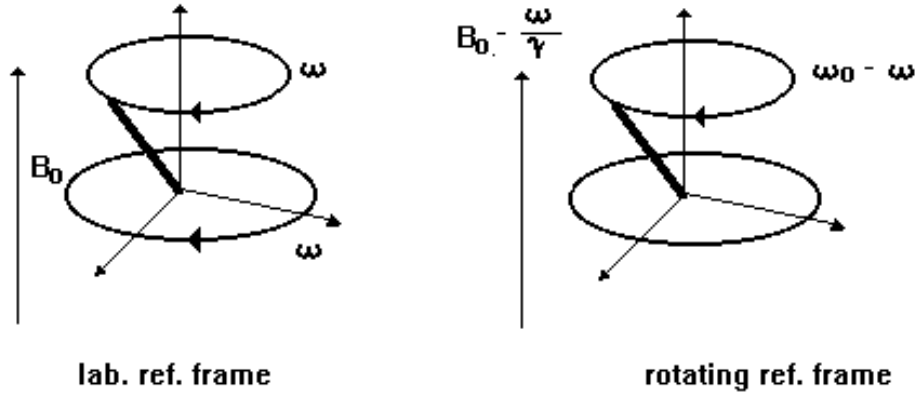
$$\begin{aligned} dM_x'/dt &= \gamma[M_y'(\omega_0 - \omega)/\gamma + B_1 M_z' \sin(\omega t)] - M_x'/T_2 \\ dM_y'/dt &= -\gamma[M_x'(\omega_0 - \omega)/\gamma + B_1 M_z' \cos(\omega t)] - M_y'/T_2 \\ dM_z'/dt &= -\gamma[-B_1 M_x' \sin(\omega t) - B_1 M_y' \cos(\omega t)] - (M_z' - M_0)/T_1 \end{aligned}$$

Viszont a $\sin(\omega t)$ tényező módosul és $\sin(\omega - \omega)$ t lesz az értéke ami éppen **0**.
Továbbá a $\cos(\omega t)$ tényező módosul és $\cos(\omega - \omega)$ t lesz az értéke, ami éppen **1**.

$$\begin{aligned} dM_x'/dt &= \gamma[M_y'(\omega_0 - \omega)/\gamma + \mathbf{0}] - M_x'/T_2 \\ dM_y'/dt &= -\gamma[M_x'(\omega_0 - \omega)/\gamma + B_1 M_z' \mathbf{1}] - M_y'/T_2 \\ dM_z'/dt &= -\gamma[\mathbf{0} - B_1 M_y' \mathbf{1}] - (M_z' - M_0)/T_1 \end{aligned}$$

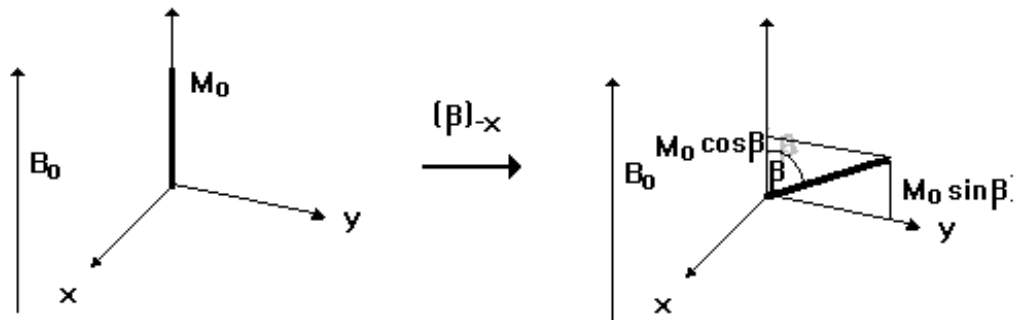
$$\begin{aligned} dM_x'/dt &= M_y'(\omega_0 - \omega) - M_x'/T_2 \\ dM_y'/dt &= -M_x'(\omega_0 - \omega) + \gamma B_1 M_z' - M_y'/T_2 \\ dM_z'/dt &= \gamma B_1 M_y' - (M_z' - M_0)/T_1 \end{aligned} \quad \text{mivel } \gamma B_1 = \omega_1$$

Thus, transforming the Bloch equation from laboratory frame (x,y,z) to the rotat. frame (x',y',z').



$$\begin{aligned}
 dM_{x'}/dt &= (\omega_0 - \omega)M_{y'} & - M_{x'}/T_2 \\
 dM_{y'}/dt &= -(\omega_0 - \omega)M_{x'} & + \omega_1 M_{z'} & - M_{y'}/T_2 \\
 dM_{z'}/dt &= -\omega_1 M_{y'} & - (M_{z'} - M_0)/T_1
 \end{aligned}$$

FREE PRECESSION (the absence of radio-frequency field)



Az így definiált egyenlet segítségével írjuk fel az \mathbf{M} precessziót arra az esetre, amikor a B_1 segédteret éppen kikapcsoltuk ($B_1=0$), tehát éppen véget ért a nem-szellektív gerjesztő pulzus:

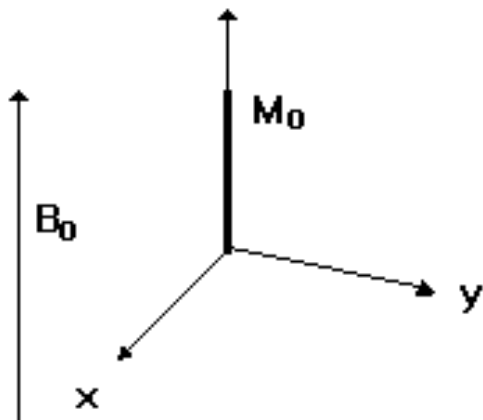
$$\begin{aligned}
 dM_{x'}/dt &= (\omega_0 - \omega)M_{y'} && - M_{x'}/T_2 \\
 dM_{y'}/dt &= -(\omega_0 - \omega)M_{x'} && + \omega_1 M_{z'} && - M_{y'}/T_2 \\
 dM_{z'}/dt &= -\omega_1 M_{y'} && - (M_{z'} - M_0)/T_1
 \end{aligned}$$

Mivel $B_1 = 0$ ezért $-\omega_1 M_{y'} = 0$ és $\omega_1 M_{z'} = 0$ tehát

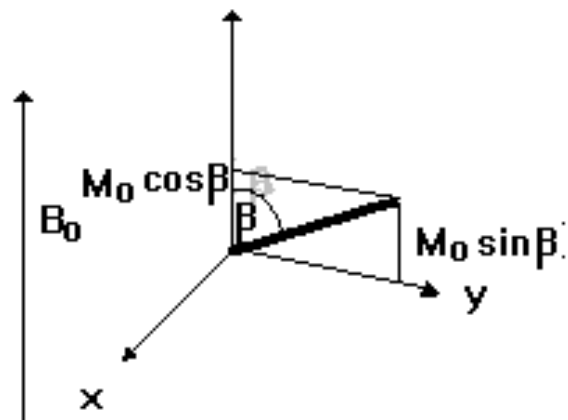
$$\begin{aligned}
 dM_{x'}/dt &= (\omega_0 - \omega)M_{y'} && - M_{x'}/T_2 \\
 dM_{y'}/dt &= -(\omega_0 - \omega)M_{x'} && - M_{y'}/T_2 \\
 dM_{z'}/dt &= - (M_{z'} - M_0)/T_1
 \end{aligned}$$

*Mielőtt megoldanánk ezt a differenciál egyenlet-rendszert,
a következőket érdemes megfontolnunk:*

$$\begin{aligned} dM_x'/dt &= (\omega_0 - \omega)M_y' & - M_x'/T_2 \\ dM_y'/dt &= -(\omega_0 - \omega)M_x' & - M_y'/T_2 \\ dM_z'/dt &= -(M_z' - M_0)/T_1 \end{aligned}$$



(β) -x
→



initial conditions after the β pulse:

$$M_z(t=0) = M_0 \cos(\beta)$$

if $\beta=90^\circ$

$$M_z(t=0) = 0$$

$$M_x(t=0) = 0$$

$$M_x(t=0) = 0$$

$$M_y(t=0) = M_0 \sin(\beta)$$

if $\beta=90^\circ$

$$M_y(t=0) = M_0$$

A: the alteration of the Z magnetization ($\beta=90^\circ$)

$$dM_z'/dt = -(M_z' - M_0)/T_1$$

memo: M_z' is toward its M_0 [equil. value] with time constant T_1 .

A szétválasztható változójú differenciálegyenlet megoldását az alábbi lépésekben foglalhatjuk össze:

$$dM_z'/(M_z' - M_0) = -1/T_1 dt \Rightarrow$$

$$(M_z' - M_0)^{-1} dM_z' = -1/T_1 dt \Rightarrow$$

$$\ln(M_z' - M_0) = -t/T_1 + \text{const.} \Rightarrow$$

$$M_z' - M_0 = C \exp(-t/T_1) \Rightarrow$$

$$M_z' = M_0 - \exp(-t/T_1)C.$$

the solution of the diff. equ.:

at $t=0$ (after the 90°_x) [$M_z(0) = M_0$]

$$M_z(t) = M_0 + \exp(-t/T_1)[M_z(t=0) - M_0]$$

$$M_z'(t) = M_0 + \exp(-t/T_1)[-M_0]$$

$$M_z(t) = M_0 (1 - \exp(-t/T_1))$$

B: the damped rotation in the X,Y plane ($\beta=90^\circ$)

$$dM_x'/dt = (\omega_0 - \omega)M_y' - M_x'/T_2$$

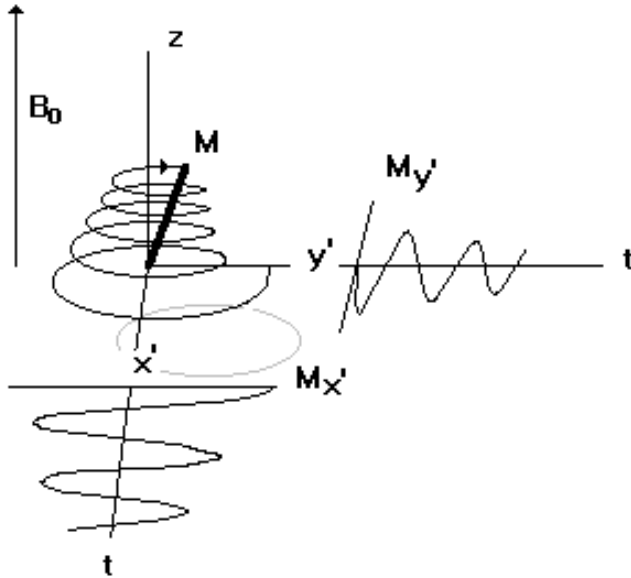
$$dM_y'/dt = -(\omega_0 - \omega)M_x' - M_y'/T_2$$

If precession starts at $t=0$ (after 90°_{-x}), then $M_{x'}=0$ and $M_{y'}=M_0$

the solution of the diff. equ.

$$M_{x'}(t) = M_0 \exp(-t/T_2) \sin(\omega_0 - \omega)$$

$$M_{y'}(t) = M_0 \exp(-t/T_2) \cos(\omega_0 - \omega)$$



the damped rotation of \mathbf{M} in the x', y' plane induces the oscillation of the $\mathbf{M}_{y'}$ and $\mathbf{M}_{x'}$.

memo : The two components are 90° out of phase.

If $M_{x'}$ is the real and $M_{y'}$ is the imaginary part of the complex magnetization, then:

$$M^+ = M_{x'} + i M_{y'}$$

therefore:

$$\begin{aligned}
 dM_{x'} / dt &= (\omega_0 - \omega)M_{y'} - M_{x'}/T_2 \\
 + d(iM_{y'})/dt &= -i(\omega_0 - \omega)M_{x'} - iM_{y'}/T_2 \\
 \hline
 dM^+ / dt &= (\omega_0 - \omega)(M_{y'} - iM_{x'}) - M^+/T_2
 \end{aligned}$$

$$dM^+ / dt = (\omega_0 - \omega)(M_{y'} - iM_{x'}) - M^+/T_2 \quad / (1 = -i^2)$$

$$dM^+ / dt = (\omega_0 - \omega)(-i^2M_{y'} - iM_{x'}) - M^+/T_2$$

$$dM^+ / dt = -i(\omega_0 - \omega)(iM_{y'} + M_{x'}) - M^+/T_2$$

$$dM^+ / dt = -i(\omega_0 - \omega)(M^+) - M^+/T_2$$

$$dM^+ / dt = -\{i(\omega_0 - \omega) + 1/T_2\}(M^+)$$

Röviden tehát a

$$dM^+ / dt = -\{i(\omega_0 - \omega) + 1/T_2\}(M^+)$$

differenciál egyenlet megoldás ---> $M^+(t) = M^+_0 \exp \{-[i(\omega_0 - \omega) + 1/T_2]t\} M^+(0)$

memo: az előbbi diff egy megoldása kifejtve:

$$dM^+ / dt = -\{i(\omega_0 - \omega) + 1/T_2\}(M^+)$$

egy szétválasztható változójú differenciálegyenlet, s amelynek megoldását

$$dM_+ / dt = -a M_+ \Rightarrow$$

$$dM_+ / M_+ = -a dt \Rightarrow$$

$$(M_+)^{-1} dM_+ = -a dt \Rightarrow$$

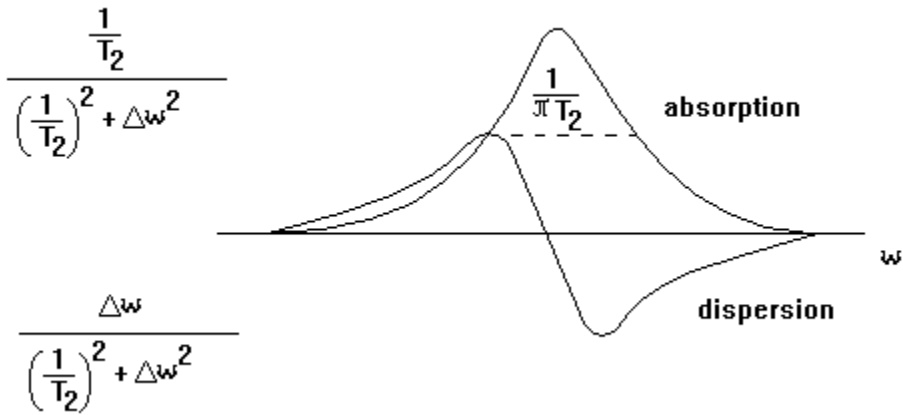
$$\ln(M_+) = -at + \text{const.} \Rightarrow$$

$$M_+ = C \exp(-at) \Rightarrow$$

$$M_+ = C \exp(-\{i(\omega_0 - \omega) + 1/T_2\}t)$$

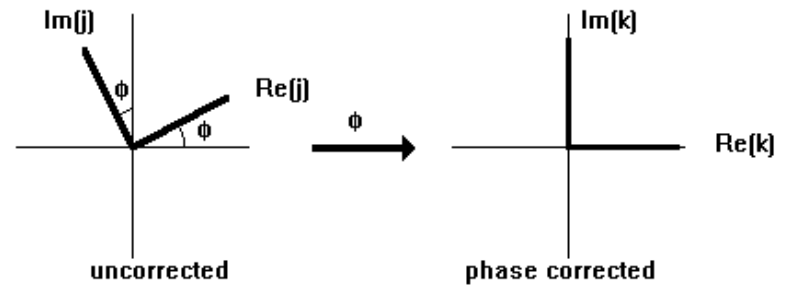
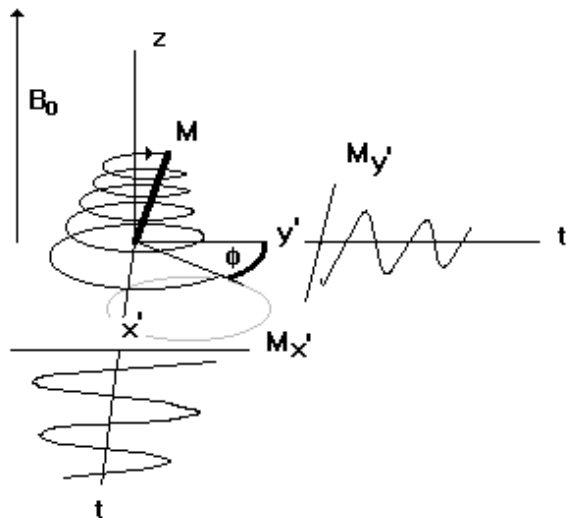
the solution of the diff. equ. --->

$$M^+(t) = M^+_0 \exp \{-[i(\omega_0 - \omega) + 1/T_2]t\} M^+(0)$$



The phasing of the solution

Since there is a time delay between the r.f. pulse and the $t=0$ of the acquisition (instrumental reason) a phase correction (ϕ) is needed.



THE STEADY-STATE AND THE TRANSIENT SOLUTION OF THE BLOCH EQUATIONS

$$M_{x'} = M_o(2\pi[\omega_o - \omega]\gamma B_1 T_2^2) / \{1 + (2\pi[\omega_o - \omega]T_2^2) + \gamma^2 B_1^2 T_1 T_2\}$$

$$M_{y'} = M_o(\gamma B_1 T_2) / \{1 + (2\pi[\omega_o - \omega]T_2^2) + \gamma^2 B_1^2 T_1 T_2\}$$

$$M_{z'} = M_o(4\pi^2[\omega_o - \omega]^2 T_2^2) / \{1 + (2\pi[\omega_o - \omega]T_2^2) + \gamma^2 B_1^2 T_1 T_2\}$$

if $T_1 = T_2$

$$a = 2\pi[\omega_o - \omega]T_2$$

$$b^2 = \gamma^2 B_1^2 T_1 T_2$$

then

$$M_{x'} = M_o a b / (1+a^2+b^2) \quad \text{dispersive spec.}$$

$$M_{y'} = M_o b / (1+a^2+b^2) \quad \text{absorptive spec.}$$

$$M_{z'} = M_o(1+a^2)/(1+a^2+b^2) \quad \text{population diff.}$$

A: the transient solution : $b^2 \ll 1$ (B_1 is sufficiently low to prevent saturation)

The absorptive spec.

$$M_y = M_o b / (1+a^2) \quad \text{a pure Lorentzian.}$$

max. value:

$$\omega_o - \omega = 0 \rightarrow a = 0$$

$$M_y(\text{max.}) = M_o b / (1+b^2)$$

$$\text{if } b \ll 1 \text{ then } M_y(\text{max.}) = M_o b$$

half-width:

$$1/2 M_o b = M_o b / (1+a^2)$$

$$2 = (1+a^2)$$

$$1 = a^2 = (2\pi[\omega_o - \omega_{1/2}]T_2)^2$$

$$[\omega_o - \omega_{1/2}] = \pm 1/(2\pi T_2)$$

the width at half-height (half-width): $\Delta\omega_{1/2} = 1/(\pi T_2)$

B: $b^2=1$ (B_1 has a higher power)

The absorptive spec.

$$M_y = M_o \frac{1}{1+a^2+1}.$$

max. value:

$$\omega_o - \omega = 0 \rightarrow a = 0$$

$$M_y(\text{max.}) = M_o \frac{1}{1+1}$$

$$M_y(\text{max.}) = M_o / 2$$

half-width:

$$1/2(M_o/2) = M_o \frac{1}{1+a^2+1}$$

$$4 = (1+a^2+1)$$

$$2 = a^2 = (2\pi[\omega_o - \omega_{1/2}]T_2)^2$$

$$[\omega_o - \omega_{1/2}] = \pm\sqrt{2}/(2\pi T_2)$$

the width at half-height (half-width): $\Delta\omega_{1/2} = \sqrt{2}/(\pi T_2)$

conclusion : the signal is more intensive, but with a larger half-width.

C: the steady-state solution : $b^2 \gg 1$ (B_1 has a high power)

The absorptive spec.

$$M_{y'} = M_0 b / (1+a^2+b^2).$$

max. value:

$$\omega_0 - \omega = 0 \rightarrow a = 0$$

$$M_{y'}(\text{max.}) = M_0 b / (1+b^2)$$

half-width:

$$1/2 \{ M_0 b / (1+b^2) \} = M_0 b / (1+a^2+b^2)$$

$$1/2 \{ 1 / (1+b^2) \} = 1 / (1+a^2+b^2)$$

$$2 + 2b^2 = 1+a^2+b^2$$

$$1 + b^2 = a^2$$

$$1 + b^2 = a^2 = (2\pi[\omega_0 - \omega_{1/2}]T_2)^2$$

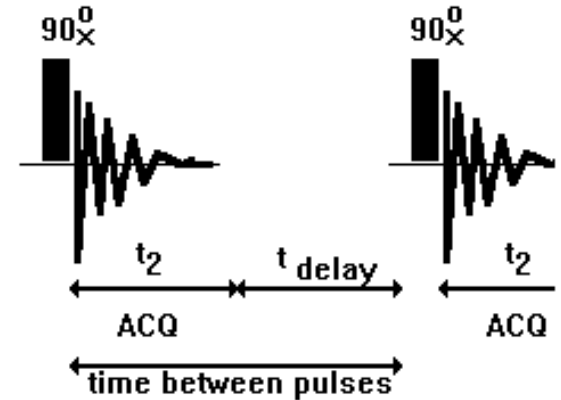
$$[\omega_0 - \omega_{1/2}] = \pm \sqrt{(1 + b^2) / (2\pi T_2)}$$

the width at half-height (half-width): $\Delta\omega_{1/2} = \sqrt{(1 + b^2) / (\pi T_2)}$

conclusion : the signal is more intensive, but with a larger half-width.

FT- and CW-NMR

If the "time between pulses" is short, then the Boltzmann distribution is not restored between pulses and saturation is reached (this is the steady-state situation).



FT-NMR (the transient solution)

(half-width): $\Delta\omega_{1/2} = 1/(\pi T_2)$

sampling time $\rightarrow n T_2$

measurement time $\rightarrow n T_2$

the total

comment :

$$F n T_2 / \Delta = N n T_2$$

CW-NMR (the steady-state solution)

(half-width): $\Delta\omega_{1/2} = \sqrt{(1 + \gamma^2 B_1^2 T_1 T_2)} / (\pi T_2)$

slow passage $\rightarrow n T_2$ where $n = 2, 3$

$$F n T_2 / \Delta$$

where F / Δ is the number of lines in sweep width

During the measurement time of a CW-NMR spectrum N FT-NMR transient can be recorded.

If $F = 10$ kHz and

$$\Delta = 1 \text{ Hz}$$

then 10^4 transient could be recorded.

If so, then sensitivity (signal to noise) improves ($\sqrt{N} = 100$)

memo : piano playing ---> CW-NMR (individual notes are presented)

---> FT-NMR (all accords are played at the same time)