

1D- and 2D-building blocks

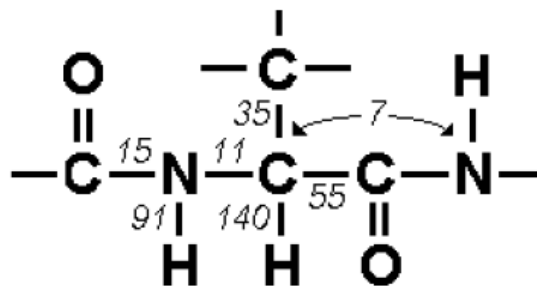
The coherence transfer pathway of a typical NMR building block.

exp.	coherence transfer	comment
spin-echo	$-I_y \Rightarrow 2I_x S_z$ $2I_x S_z \Rightarrow I_y$	$J_{IS} > 0$ and $\tau = 1/(4J_{IS})$
	$-I_y \Rightarrow -I_y$	$J_{IS} = 0$
INEPT	$-I_y \Rightarrow 2I_z S_y$	$J_{IS} > 0$
ref. INEPT	$-I_y \Rightarrow -S_y$	$J_{IS} > 0$
rev. INEPT	$2I_z S_y \Rightarrow -I_y$	$J_{IS} > 0$
ref. rev. INEPT	$-S_y \Rightarrow -I_y$	$J_{IS} > 0$
COSY	$-I_y \Rightarrow -2I_z S_y \sin(\Omega_I t) \sin(\pi J_{IS} t)$	$J_{IS} > 0$ (off diagonal)
TOCSY	$-I_y \Rightarrow S_x$	$J_{IS} = 0$ (off diagonal)
HMQC	$-I_y \Rightarrow -I_y \cos(\Omega_s t)$	$J_{IS} > 0$
HSQC	$-I_y \Rightarrow I_x \cos(\Omega_s t)$	$J_{IS} > 0$

As homonuclear coupling const. (J_{IS}) is small ≈ 10 Hz, the associated Δ and/or $\tau \approx 1/10$ Hz ≈ 100 ms, a longer delay time

As heteronuclear coupling const. (J_{IS}) is large ≈ 100 Hz, the relevant Δ and/or $\tau \approx 1/100$ Hz ≈ 10 ms, a rather short time.

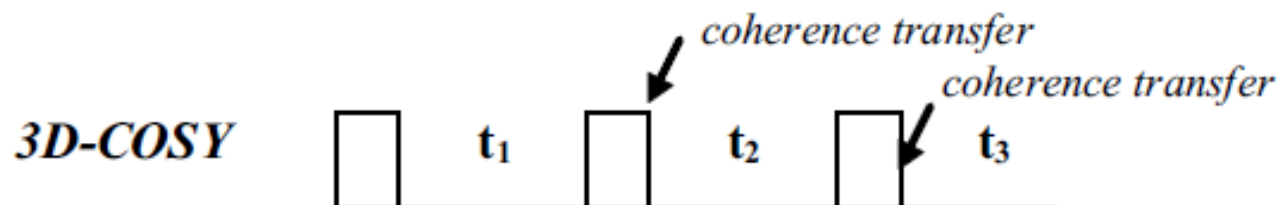
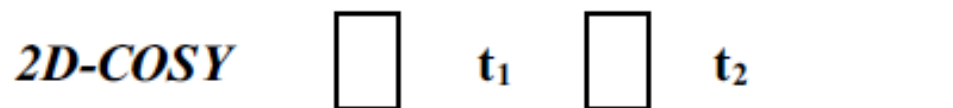
e.g. $J_{NH} \approx 90$ Hz or $J_{CH} \approx 140$ Hz



The simplest but „not too useful” 3D NMR experiment: the COSY-COSY

Homonuclear 3D NMR

Combining two *convenient* 2D experiments (e.g. 2D-COSY)



$$\sigma[\text{eq.}]$$

$$\hat{H} = \pi/2 (\hat{I}_x)$$

$$\mathbf{I}_z$$

$$\downarrow 90^\circ_x$$

AMX (ISM) spin system

$$\hat{H} = \hat{I}_z(\Omega_I t_1)$$

$$-\mathbf{I}_y$$

$$\downarrow t_1$$

$$-\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1)$$

$$\downarrow$$

$$\hat{H} = 2\hat{I}_z \check{S}_z(J_{IS}\pi t_1)$$

$$-\mathbf{I}_y \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1) + 2\mathbf{I}_x \mathbf{S}_z \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$+ \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) + 2\mathbf{I}_y \mathbf{S}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$\hat{H} = \pi/2 (\hat{I}_x + \hat{S}_z)$$

$$\downarrow 90^\circ_x$$

$$-\mathbf{I}_z \cos(\Omega_I t_1) \cos(\pi J_{IS} t_1) - 2\mathbf{I}_x \mathbf{S}_y \cos(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

$$+ \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IS} t_1) - 2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1)$$

The $-2\mathbf{I}_z \mathbf{S}_y$ term is a single quan. S coher. originating from spin I. Thus the evolution of this term is followed:

$$\hat{H} = \hat{S}_z(\Omega_S t_2)$$

$$\downarrow t_2$$

$$-2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) + 2\mathbf{I}_z \mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2)$$

$$\hat{H} = 2\hat{S}_z M_z(J_{IS}\pi t_1)$$

$$\downarrow$$

$$-2\mathbf{I}_z \mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2)$$

$$+ 4\mathbf{I}_z \mathbf{S}_x M_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)$$

$$+ 2\mathbf{I}_z \mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2)$$

$$+ 4\mathbf{I}_z \mathbf{S}_y M_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)$$

$$\begin{aligned}
& -2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2) \\
& +4\mathbf{I}_z\mathbf{S}_x\mathbf{M}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)
\end{aligned}$$

$$\begin{aligned}
& +2\mathbf{I}_z\mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2) \\
& +4\mathbf{I}_z\mathbf{S}_y\mathbf{M}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)
\end{aligned}$$

$$\hat{H} = \pi/2 (\hat{S}_x + \mathbf{M}_X) \quad \downarrow 90^\circ_x$$

$$\begin{aligned}
& -2\mathbf{I}_z\mathbf{S}_z \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2) \\
& -4\mathbf{I}_z\mathbf{S}_x\mathbf{M}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)
\end{aligned}$$

$$\begin{aligned}
& +2\mathbf{I}_z\mathbf{S}_x \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \cos(\Omega_S t_2) \cos(\pi J_{SM} t_2) \\
& -4\mathbf{I}_z\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)
\end{aligned}$$

The $-4\mathbf{I}_z\mathbf{S}_z\mathbf{M}_y$ term is a single quan. M coher. originating from spin I passed through S.
Thus the evolution of this term is followed:

$$-4\mathbf{I}_z\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1) \sin(\pi J_{IS} t_1) \sin(\Omega_S t_2) \sin(\pi J_{SM} t_2)$$

During ACQ:

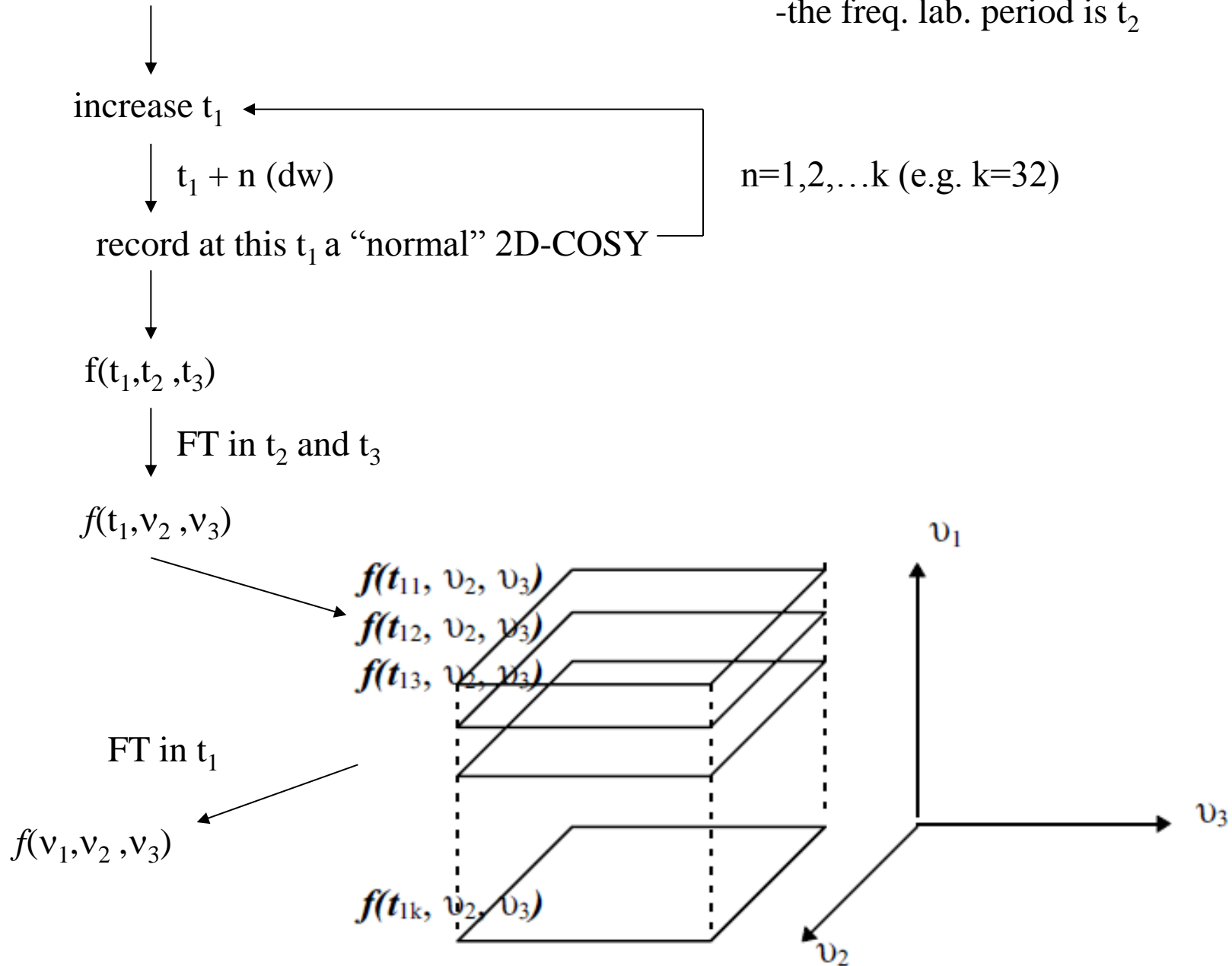
$$\hat{H} = \mathbf{M}_z(\Omega_M t_3)$$

$$\downarrow t_3$$

$$\hat{H} = 2\hat{S}_z\mathbf{M}_z(J_{SM}\pi t_3)$$

$$\downarrow$$

explanation: record at a fix t_1 a “normal” 2D-COSY where -the acquisition dim is t_3
-the freq. lab. period is t_2

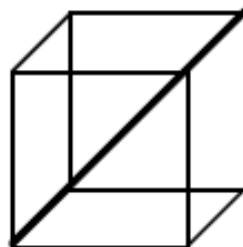


The $f(\nu_1, \nu_2, \nu_3)$ spectrum (e.g. AMX spin system)

1. Diagonal peaks [e.g. $\Omega_A(\nu_1)$, $\Omega_A(\nu_2)$, $\Omega_A(\nu_3)$] along the body diagonal:
from lower left-hand
to upper right-hand

Comment: unmigrated magnetisation during t_1 , t_2 as well as t_3

Analogue: normal 2D-COSY diagonal

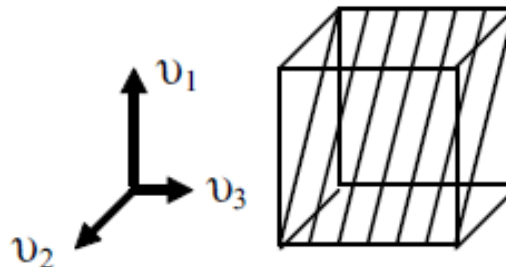


2. Cross-diagonal peaks along the cross-diagonal planes:

e.g. $\Omega_A(\nu_1)$, $\Omega_A(\nu_2)$, $\Omega_M(\nu_3)$ $\nu_1 = \nu_2$

Comment: unmigrated magnetisation during the pulse separating t_1 and t_2
than coherence is transferred during the pulse separating t_2 and t_3

Analogue: normal 2D-COSY

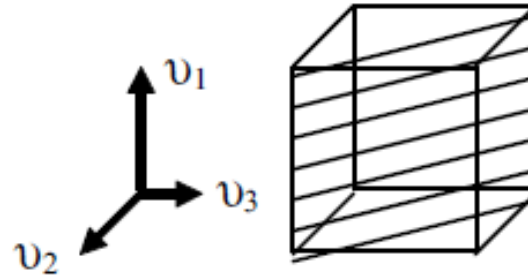


e.g. $\Omega_A(\nu_1)$, $\Omega_M(\nu_2)$, $\Omega_M(\nu_3)$

$$\nu_2 = \nu_3$$

Comment: coherence is transferred during the pulse separating t_1 and t_2
than coherence is untransferred during the pulse separating t_2 and t_3

Analogue: normal 2D-COSY



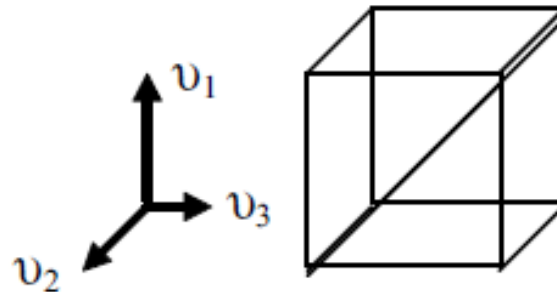
3. Back transfer peaks along the back transfer plane:

e.g. $\Omega_A(\nu_1)$, $\Omega_M(\nu_2)$, $\Omega_A(\nu_3)$

$$\nu_1 = \nu_3$$

Comment: coherence is transferred during the pulse separating t_1 and t_2
than coherence is back transferred during the pulse separating t_2 and t_3

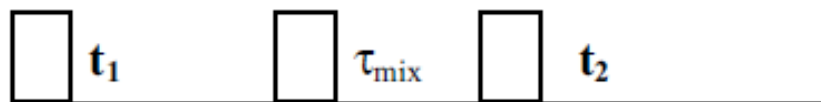
Analogue: none



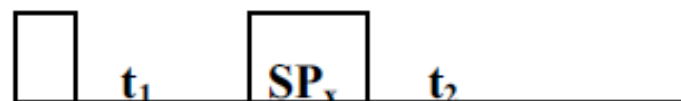
3D NOESY-TOCSY

Combining two *convenient* 2D experiments (e.g. 2D-NOESY and a 2D-TOCSY)

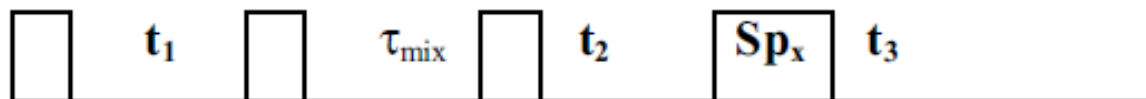
2D-NOESY



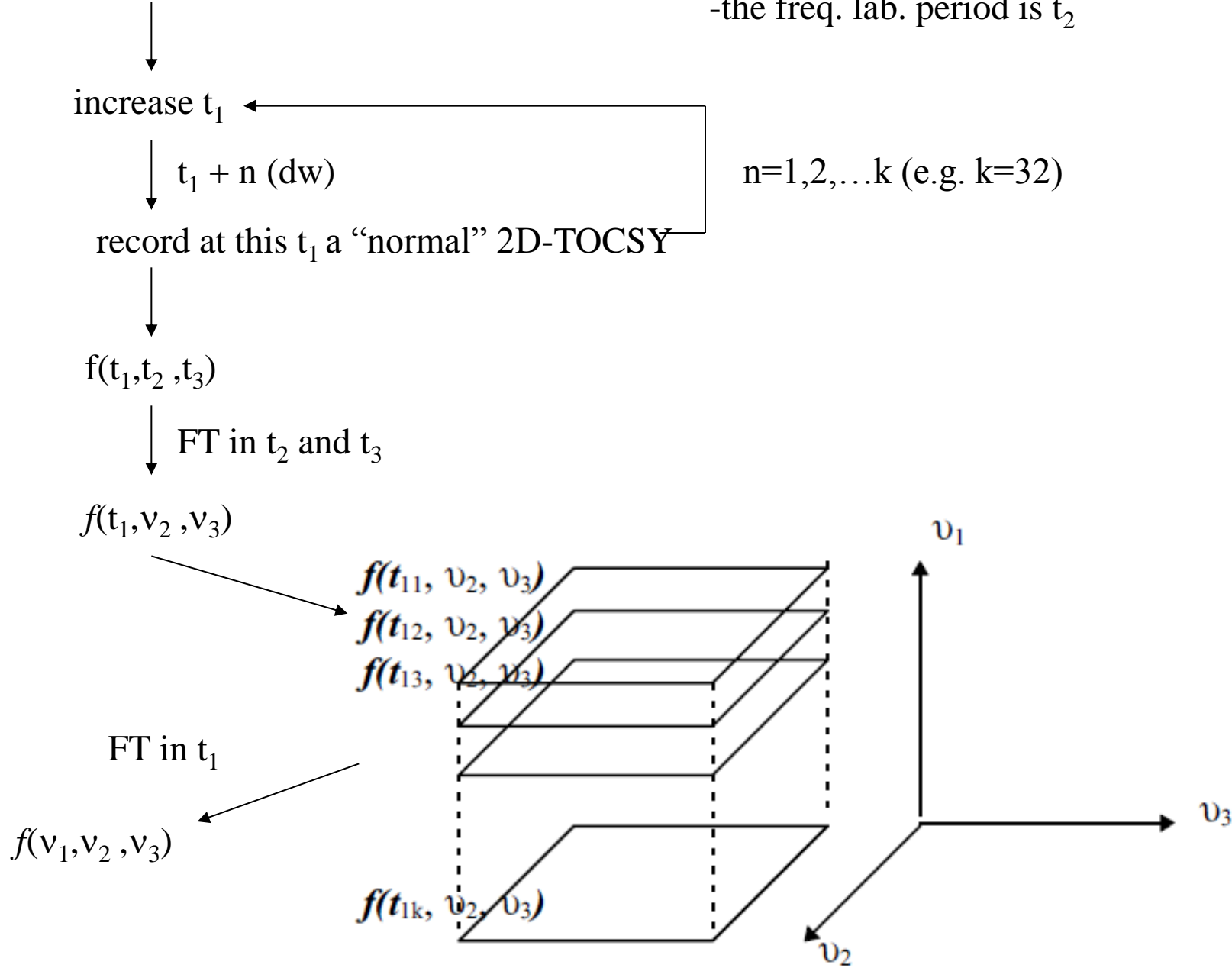
2D-TOCSY

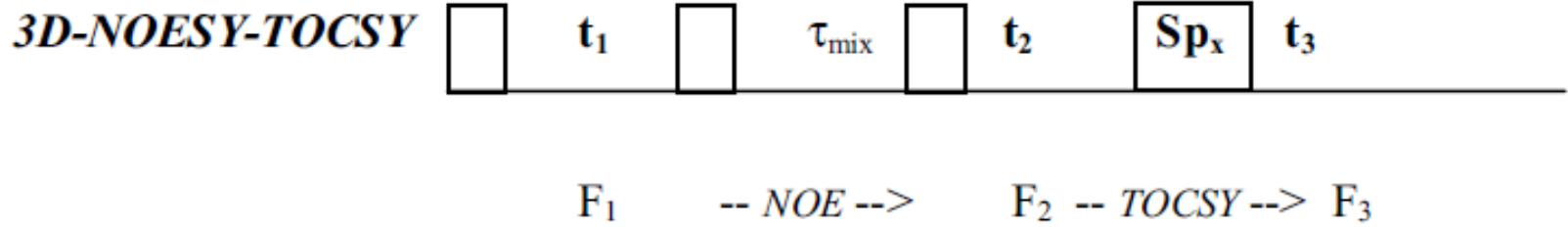


3D-NOESY-TOCSY



explanation: record at a fix t_1 a "normal" 2D-TOCSY where -the acquisition dim is t_3
-the freq. lab. period is t_2



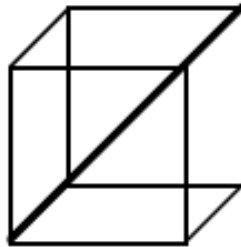


The $f(\nu_1, \nu_2, \nu_3)$ spectrum (e.g. AMX spin system)

1. Diagonal peaks [e.g. $\Omega_A(\nu_1), \Omega_A(\nu_2), \Omega_A(\nu_3)$] along the body diagonal:
from lower left-hand
to upper right-hand

Comment: unmigrated magnetisation during t_1, t_2 as well as t_3

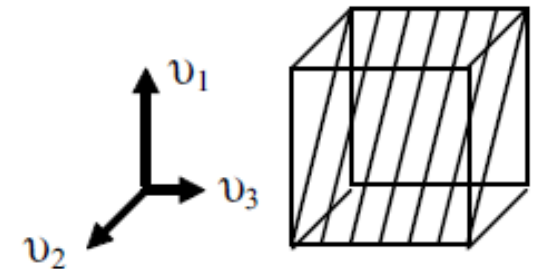
Analogue: normal 2D diagonal



2. Cross-diagonal peaks along the cross-diagonal planes:

e.g. $\Omega_A(\nu_1), \Omega_A(\nu_2), \Omega_M(\nu_3)$

$$\nu_1 = \nu_2$$



Comment: unmigrated magnetisation during the pulse separating t_1 and t_2
than coherence is transferred during the pulse separating t_2 and t_3

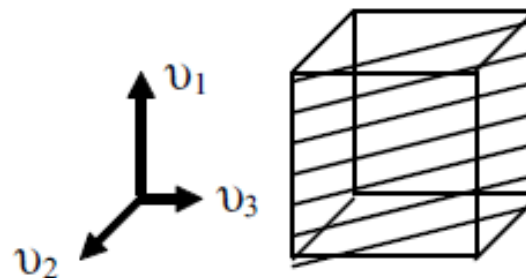
Analogue: normal 2D-TOCSY

e.g. $\Omega_A(\nu_1)$, $\Omega_M(\nu_2)$, $\Omega_M(\nu_3)$

$$\nu_2 = \nu_3$$

Comment: coherence is transferred during the pulse separating t_1 and t_2
than coherence is untransferred during the pulse separating t_2 and t_3

Analogue: normal 2D-TOCSY



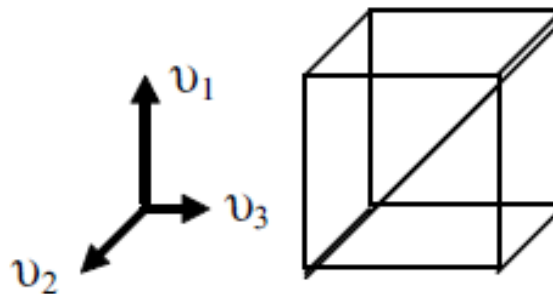
3. Back transfer peaks along the back transfer plane:

e.g. $\Omega_A(\nu_1)$, $\Omega_M(\nu_2)$, $\Omega_A(\nu_3)$

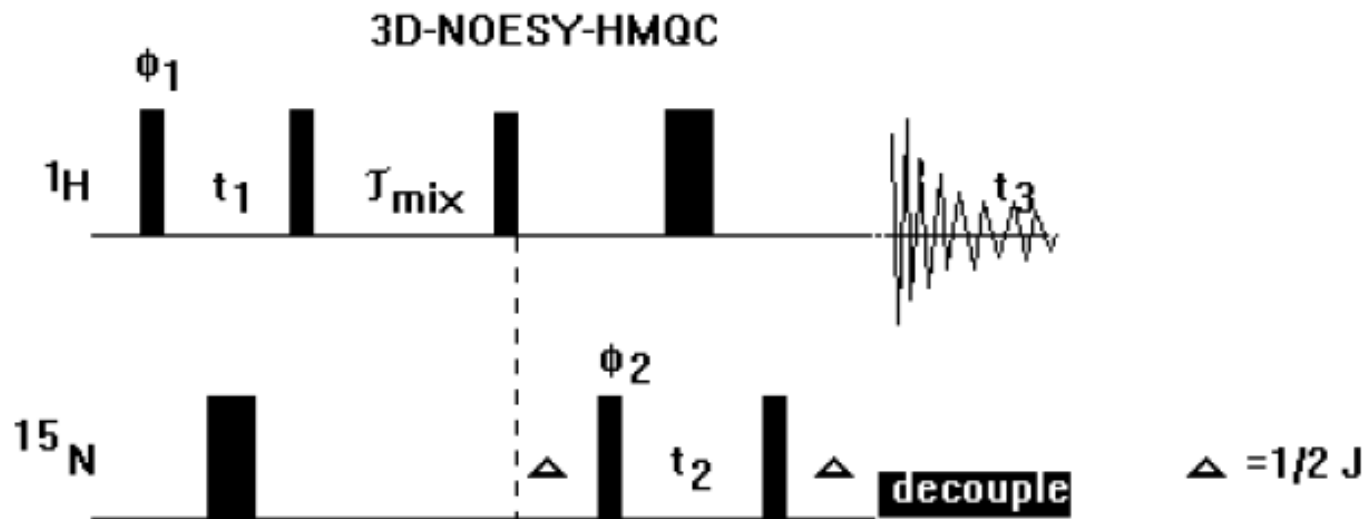
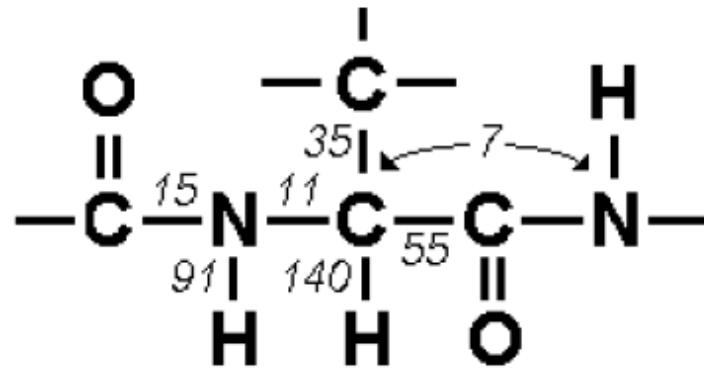
$$\nu_1 = \nu_3$$

Comment: coherence is transferred during the pulse separating t_1 and t_2
than coherence is back transferred during the pulse separating t_2 and t_3

Analogue: none



Heteronuclear 3D NMR



1. The first $90^\circ \Rightarrow$ transverse magnetisation
2. This is frequency labelled during t_1
3. The second $90^\circ \Rightarrow$ longitudinal magnetisation
4. During the mixing time \Rightarrow longitudinal magnetisation is transferred via cross-relaxation
5. The third $90^\circ \Rightarrow$ transverse magnetisation *(end of NOESY part)*

$$H_z^A \xrightarrow{90^\circ \text{H}} H_y^A \xrightarrow{t_1} H_y^A(\Omega_A t_1) \xrightarrow{90^\circ \text{H}} H_z^A(\Omega_A t_1) \xrightarrow{\text{NOE}} H_z^B(\Omega_A t_1) \xrightarrow{90^\circ \text{H}} H_y^B(\Omega_A t_1)$$

6. During the Δ time \Rightarrow anti phase coherence is created ($\Delta_{\text{ideal}} = 1/(2*91 \text{ Hz}) \approx 5 \text{ ms}$)
7. The fourth 90° (first 90° on N) \Rightarrow generates multiple quantum coherence
8. This is frequency labelled during t_2
9. The fifth 90° (second 90° on N) \Rightarrow generates from multiple quantum coherence anti phase coherence
10. During the second $\Delta \Rightarrow$ anti phase coherence is refocused to in-phase ^1H magnetisation
11. Detected during t_3 *(end of HMQC part)*

$$H_y^B(\Omega_A t_1) \xrightarrow{\Delta} H_x^B N_z(\Omega_A t_1) \xrightarrow{90^\circ \text{N}} H_x^B N_y(\Omega_A t_1) \xrightarrow{t_2} H_x^B N_y(\Omega_A t_1)(\Omega_N t_2) \xrightarrow{90^\circ \text{N}} H_x^B N_z(\Omega_A t_1)(\Omega_N t_2) \xrightarrow{\Delta} H_y^B(\Omega_A t_1)(\Omega_N t_2) \xrightarrow{t_3} H_y^B(\Omega_A t_1)(\Omega_N t_2)(\Omega_B t_3)$$

Summary:

$$H_z^A \xrightarrow{\text{3D NOESY-HMQC}} H_y^B(\Omega_A t_1)(\Omega_N t_2)(\Omega_B t_3)$$