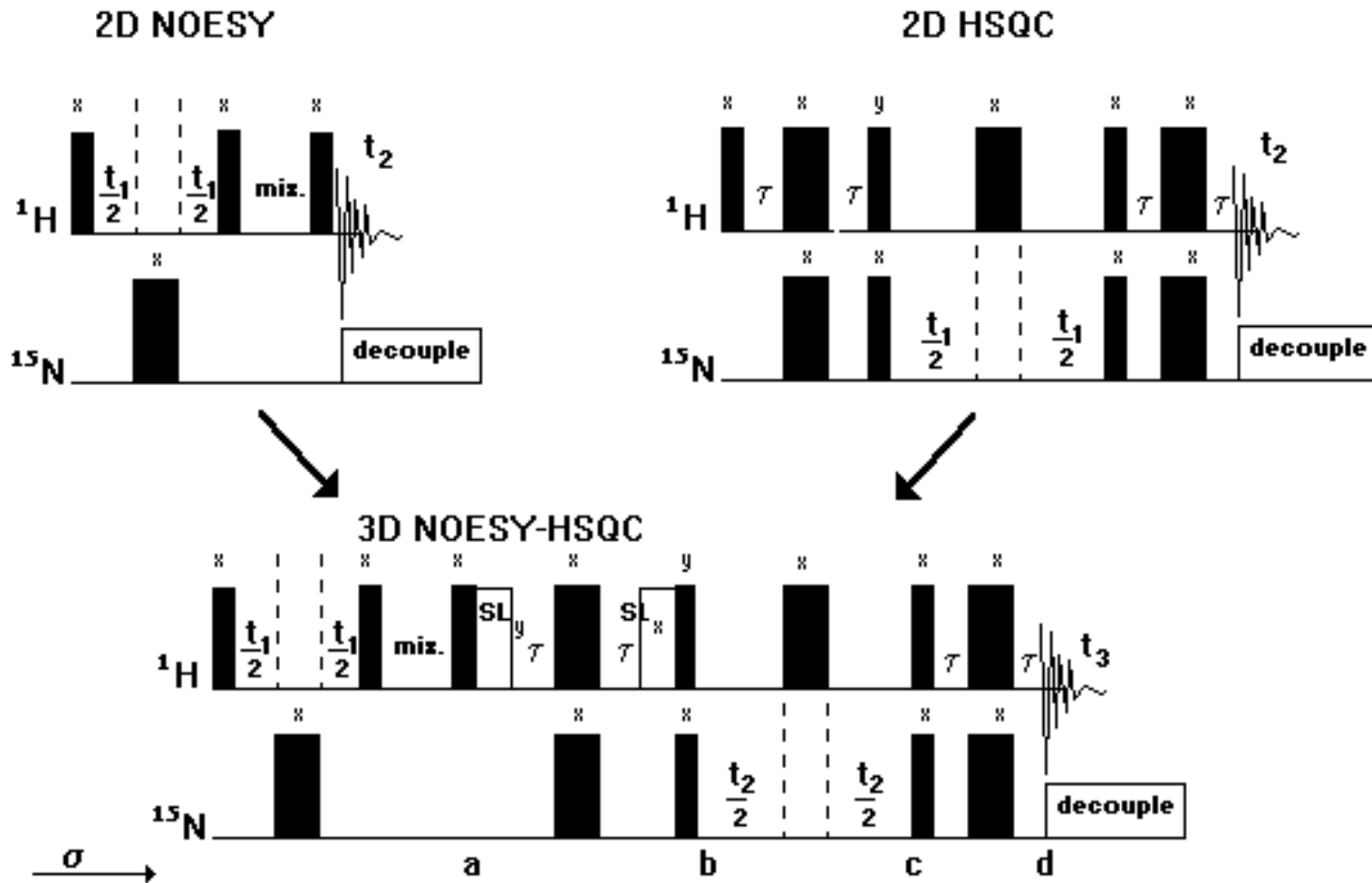


3D NOESY-HSQC



Consider:

homonuclear spins I, K with chemical shift Ω_I , Ω_K and coupling J_{IK}
 (e.g. H^{NH} and H^α)

heteronuclear spin N with chemical shift Ω_N with a coupling J_{IN}
 (e.g. H^{NH} and N^{NH})

A. module (NOESY with decoupled ^{15}N)

$\sigma[0]$ "The first step is 90°

and a heteronuclear echo module removes the heteronuclear coupling during t_1 "

$\sigma[\text{eq.}]$

$$\hat{H} = \pi/2 (\hat{I}_x + \mathbf{K}_x)$$

$\downarrow 90^\circ_x$

\mathbf{I}_z and \mathbf{K}_z

$\sigma[0]$

$$\hat{H} = \hat{I}_z(\Omega_I t_1) + \mathbf{K}_z(\Omega_K t_1)$$

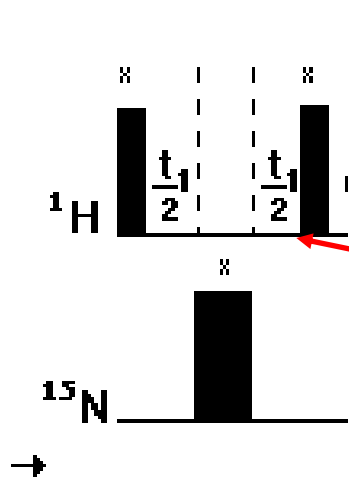
$-\mathbf{I}_y$ and $-\mathbf{K}_y$

$\downarrow t_1$

$$-\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1)$$

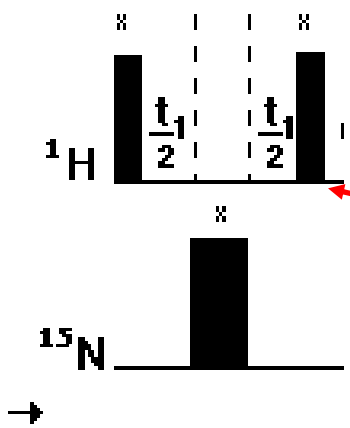
$$-\mathbf{K}_y \cos(\Omega_K t_1) + \mathbf{K}_x \sin(\Omega_K t_1)$$

$$\hat{H} = 2\hat{I}_z \mathbf{K}_z (J_{IK} \pi t_1)$$



$$\begin{aligned}
 &-\mathbf{I}_y \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 &\quad + 2\mathbf{I}_x \mathbf{K}_z \cos(\Omega_I t_1) \sin(\pi J_{IK} t_1) \\
 &+ \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 &\quad + 2\mathbf{I}_y \mathbf{K}_z \sin(\Omega_I t_1) \sin(\pi J_{IK} t_1) \\
 &-\mathbf{K}_y \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \\
 &\quad + 2\mathbf{K}_x \mathbf{I}_z \cos(\Omega_K t_1) \sin(\pi J_{IK} t_1) \\
 &+ \mathbf{K}_x \sin(\Omega_K t_1) \cos(\pi J_{IK} t_1) \\
 &\quad + 2\mathbf{I}_y \mathbf{I}_z \sin(\Omega_K t_1) \sin(\pi J_{IK} t_1)
 \end{aligned}$$

$$\hat{H} = \pi/2 (\hat{I}_x + K_x)$$



$$\downarrow 90^\circ_x \text{ memo.} = \cos(\pi/2) = 0, \sin(\pi/2) = 1$$

$$\begin{aligned}
 & -\mathbf{I}_z \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 & \quad - 2\mathbf{I}_x \mathbf{K}_y \cos(\Omega_I t_1) \sin(\pi J_{IK} t_1) \\
 & + \mathbf{I}_x \sin(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 & \quad - 2\mathbf{I}_z \mathbf{K}_y \sin(\Omega_I t_1) \sin(\pi J_{IK} t_1) \\
 & - \mathbf{K}_z \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \\
 & \quad - 2\mathbf{K}_x \mathbf{I}_y \cos(\Omega_K t_1) \sin(\pi J_{IK} t_1) \\
 & + \mathbf{K}_x \sin(\Omega_K t_1) \cos(\pi J_{IK} t_1) \\
 & \quad - 2\mathbf{K}_z \mathbf{I}_y \sin(\Omega_K t_1) \sin(\pi J_{IK} t_1)
 \end{aligned}$$

memo.1: selecting only z magnetization ($-\mathbf{I}_z$ and $-\mathbf{K}_z$)

$\sigma[t_1, 0]$

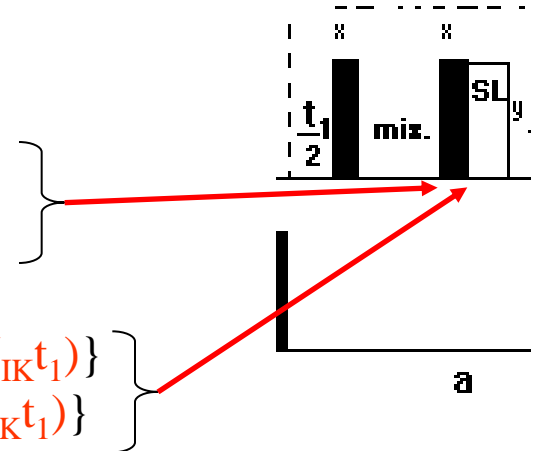
$$\begin{aligned}
 & -\mathbf{I}_z \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 & -\mathbf{K}_z \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1)
 \end{aligned}$$

since during the mixing time (τ_m) only the populations ($-\mathbf{I}_z a_{II}$, $-\mathbf{I}_z a_{IK}$, $-\mathbf{I}_z a_{KI}$ and $-\mathbf{I}_z a_{KK}$) interact, where the interaction or mixing coefficients are : a_{II} , a_{IK} , a_{KI} , a_{KK})

after the mixing the populations are:

$$\begin{aligned}
 & -\mathbf{I}_z a_{II} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) & -\mathbf{I}_z a_{IK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \\
 & -\mathbf{K}_z a_{KK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) & -\mathbf{K}_z a_{KI} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \\
 & \hat{H} = \pi/2 (\hat{I}_x + K_x) & \downarrow 90^\circ_x
 \end{aligned}$$

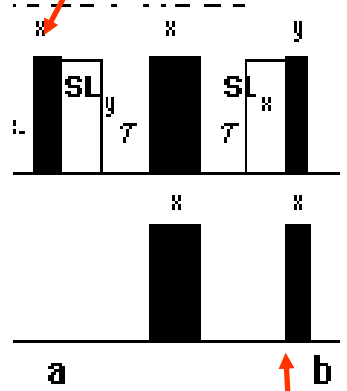
$$\begin{aligned}
 +\mathbf{I}_y & \{ a_{II} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) & + a_{IK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \} \\
 +\mathbf{K}_y & \{ a_{KK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) & + a_{KI} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \}
 \end{aligned}$$



$\sigma[a]$	$+I_y$	$\{ a_{II} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \}$	$+ a_{IK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \}$
	$+K_y$	$\{ a_{KK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \}$	$+ a_{KI} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \}$

B. generating anti-phase magnetization on ^{15}N

memo.2: The x and y spin-locks are present for water suppression.
(e.g. on a D_2O sample it would be not needed.)



$\sigma[a]$ "The first INEPT module"

$\hat{H} = \text{echo (homo)}$

$\hat{H} = I_y(\pi/2)$

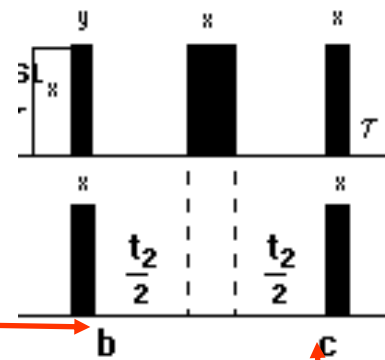
$\hat{H} = N_x(\pi/2)$

$$\begin{aligned}
 & \downarrow \quad +I_y \\
 & \{-I_y \cos(J_{HN} \pi 2\tau) + 2I_x N_z \sin(J_{HN} \pi 2\tau) \text{ with } 2\tau = 1/2J_{IN}\} \\
 & \quad + 2I_x N_z \\
 & \quad \downarrow \\
 & \quad -2I_z N_z \\
 & \quad \downarrow \\
 & \quad +2I_z N_y
 \end{aligned}$$

$\sigma[b]$ "at the end of the INEPT module"

$+2I_z N_y$	$\{ a_{II} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \}$	$+ a_{IK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \}$
$+2K_y N_y$	$\{ a_{KK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \}$	$+ a_{KI} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \}$

C. module (frequency labelling by N)



$\sigma[a] \rightarrow \sigma[b]$ "frequency labelling by ^{15}N "
memo.3: echo decouples from N all protons

$\sigma[b]$ "at the beginning of t_2 "

$$+2\mathbf{I}_z\mathbf{N}_y$$

\hat{H} = echo (hetero)

$\sigma[c]$ "at the end of t_2 "

$$\downarrow$$

$$-2\mathbf{I}_z\mathbf{N}_y\cos(\Omega_N t_2) + 2\mathbf{I}_z\mathbf{N}_x\sin(\Omega_N t_2)$$

memo.4 : $-2\mathbf{I}_z\mathbf{N}_x\sin(\Omega_N t_2)$ is phase cycled out (in a gradient enhanced version it is kept.)

$\sigma[c]$ "at the end of the frequency labelling module"

$$-2\mathbf{I}_z\mathbf{N}_y \quad * \cos(\Omega_N t_2) * \{ a_{\text{II}} \cos(\Omega_I t_1) \cos(\pi J_{\text{IK}} t_1) \quad + a_{\text{IK}} \cos(\Omega_K t_1) \cos(\pi J_{\text{IK}} t_1) \}$$

$$-2\mathbf{K}_y\mathbf{N}_y \quad * \cos(\Omega_N t_2) * \{ a_{\text{KK}} \cos(\Omega_K t_1) \cos(\pi J_{\text{IK}} t_1) \quad + a_{\text{KI}} \cos(\Omega_I t_1) \cos(\pi J_{\text{IK}} t_1) \}$$

D. module (finishing with a reverse INEPT)

$\sigma[g]$ "The reverse INEPT module"

$$\hat{H} = I_x(\pi/2)$$

$$\hat{H} = N_x(\pi/2)$$

$$\hat{H} = \text{echo (homo)}$$

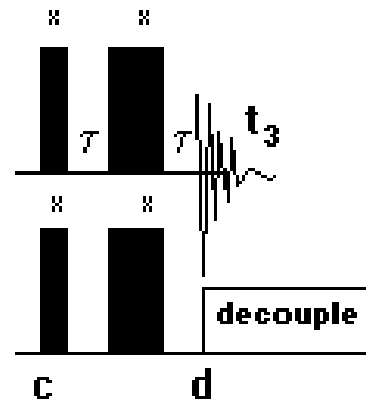
$$-2I_z N_y$$



$$+2I_y N_y$$



$$+2I_y N_z$$



$$\{-I_x \sin(J_{HN}\pi 2\tau) + 2I_y N_z \cos(J_{IN}\pi 2\tau) \text{ with } 2\tau = 1/2J_{IN}\}$$

$$\sigma[h] \text{ " at the end": } \begin{matrix} -I_x & * \cos(\Omega_N t_2) * \{ a_{II} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) & + a_{IK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \} \\ -K_x & * \cos(\Omega_N t_2) * \{ a_{KK} \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) & + a_{KI} \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \} \end{matrix}$$

$$ACQ: \{ \hat{H} = I_z(\Omega_I[t_3]) \text{ and } K_z(\Omega_K[t_3]) \}$$

memo.5: heteronuclear coupling is not affective during acquisition
but homonuclear coupling J_{IK} is present.

memo.6: The four terms of $\sigma[h]$ evolves during ACQ into 16 terms (see NOESY).
Putting the receiver on x the above 4 terms remains:

$$\begin{matrix} -I_x a_{II} & \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \sin(\Omega_I t_3) \cos(\pi J_{IK} t_3) * \cos(\Omega_N t_2) \\ -K_x a_{KK} & \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \sin(\Omega_K t_3) \cos(\pi J_{IK} t_3) * \cos(\Omega_N t_2) \\ -I_x a_{IK} & \cos(\Omega_K t_1) \cos(\pi J_{IK} t_1) \sin(\Omega_I t_3) \cos(\pi J_{IK} t_3) * \cos(\Omega_N t_2) \\ -K_x a_{KI} & \cos(\Omega_I t_1) \cos(\pi J_{IK} t_1) \sin(\Omega_K t_3) \cos(\pi J_{IK} t_3) * \cos(\Omega_N t_2) \end{matrix}$$

$$\text{memo.7: } \sin(A)\cos(B) = 1/2[\sin(A+B)+\sin(A-B)]$$

$$\cos(A)\cos(B) = 1/2[\cos(A+B)+\cos(A-B)]$$

therefore:

$$\begin{aligned}
 -1/4\mathbf{I}_x a_{II} & \quad [+\cos\{(\Omega_I+\pi J_{IK})t_1\} + \cos\{(\Omega_I-\pi J_{IK})t_1\}]^* \\
 & \quad [+\sin\{(\Omega_I+\pi J_{IK})t_3\} + \sin\{(\Omega_I-\pi J_{IK})t_3\}]^* \cos(\Omega_N t_2) \\
 -1/4\mathbf{K}_x a_{KK} & \quad [+\cos\{(\Omega_K+\pi J_{IK})t_1\} + \cos\{(\Omega_K-\pi J_{IK})t_1\}]^* \\
 & \quad [+\sin\{(\Omega_K+\pi J_{IK})t_3\} + \sin\{(\Omega_K-\pi J_{IK})t_3\}]^* \cos(\Omega_N t_2) \\
 -1/4\mathbf{I}_x a_{IK} & \quad [+\cos\{(\Omega_K+\pi J_{IK})t_1\} + \cos\{(\Omega_K-\pi J_{IK})t_1\}]^* \\
 & \quad [+\sin\{(\Omega_I+\pi J_{IK})t_3\} + \sin\{(\Omega_I-\pi J_{IK})t_3\}]^* \cos(\Omega_N t_2) \\
 -1/4\mathbf{K}_x s_{KI} & \quad [+\cos\{(\Omega_I+\pi J_{IK})t_1\} + \cos\{(\Omega_I-\pi J_{IK})t_1\}]^* \\
 & \quad [+\sin\{(\Omega_K+\pi J_{IK})t_3\} + \sin\{(\Omega_K-\pi J_{IK})t_3\}]^* \cos(\Omega_N t_2)
 \end{aligned}$$

the following terms can be found

$$\begin{aligned}
 -\mathbf{I}_x a_{II} & \quad [+ \dots + \dots + \dots + \dots] \text{ at } \Omega_I, \Omega_I \\
 -\mathbf{I}_x a_{IK} & \quad [+ \dots + \dots + \dots + \dots] \text{ at } \Omega_K, \Omega_I \\
 -\mathbf{K}_x a_{KK} & \quad [+ \dots + \dots + \dots + \dots] \text{ at } \Omega_K, \Omega_K \\
 -\mathbf{K}_x a_{KI} & \quad [+ \dots + \dots + \dots + \dots] \text{ at } \Omega_I, \Omega_K
 \end{aligned}$$

setting the phase such as

cos is absorptive (a) in t_1

sin is absorptive (d) in t_2

cos is absorptive (a) in t_3

$$\begin{aligned}
 \mathbf{I}_x a_{II} & \quad [+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_I, \Omega_I \text{ and } [+a] \text{ in } \Omega_N \\
 \mathbf{I}_x a_{IK} & \quad [+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_K, \Omega_I \text{ and } [+a] \text{ in } \Omega_N \\
 \mathbf{K}_x a_{KK} & \quad [+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_K, \Omega_K \text{ and } [+a] \text{ in } \Omega_N \\
 \mathbf{K}_x a_{KI} & \quad [+a \dots +a \dots +a \dots +a \dots] \text{ at } \Omega_I, \Omega_K \text{ and } [+a] \text{ in } \Omega_N
 \end{aligned}$$

Conclusions (e.g. $\{^1\text{H}-^{15}\text{N}\}$ 3D NOESY-HSQC):

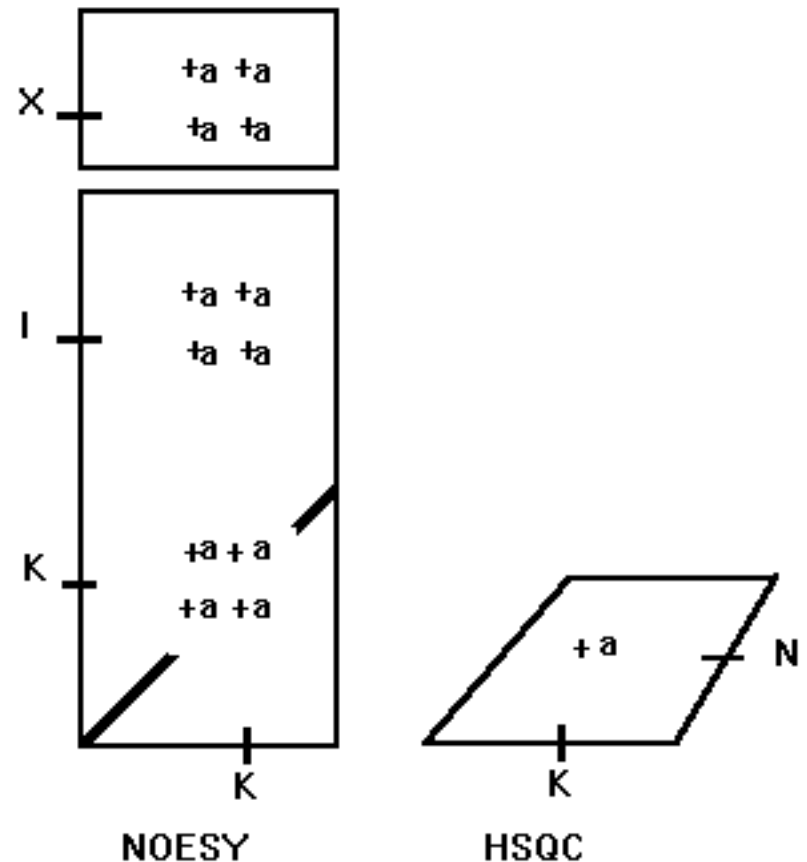
memo.8: homonuclear spins I, K with chemical shift Ω_I , Ω_K and coupling J_{IK} (e.g. H^{NH} and H^α) were involved in the analysis.

Because of the HSQC module of the experiment the NH region [$\text{H}_{\text{NH}} \{ \approx 7.2 \pm 2.5 \text{ ppm} \}$ and $\text{N} \{ \approx 117 \pm 20 \text{ ppm} \}$] is detected therefore only the "left" region of the NOESY is only present.

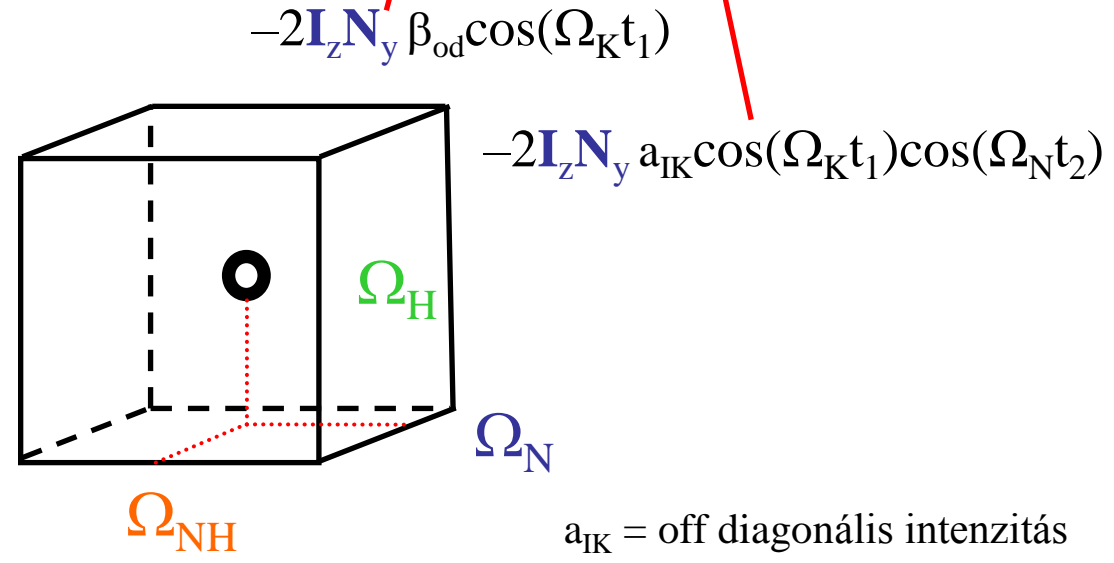
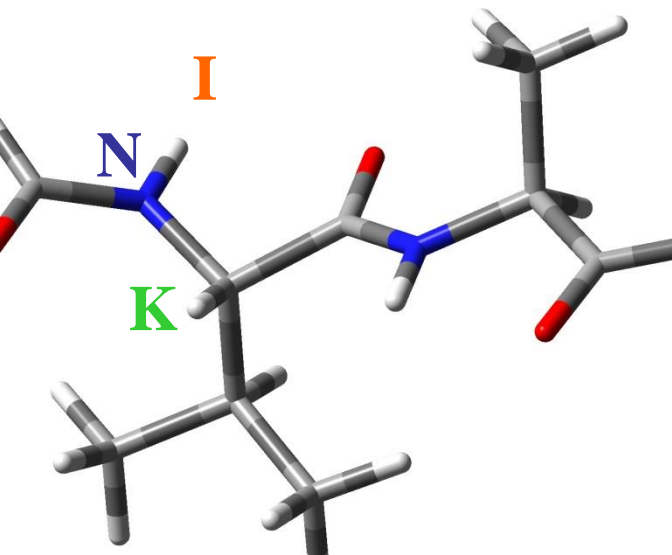
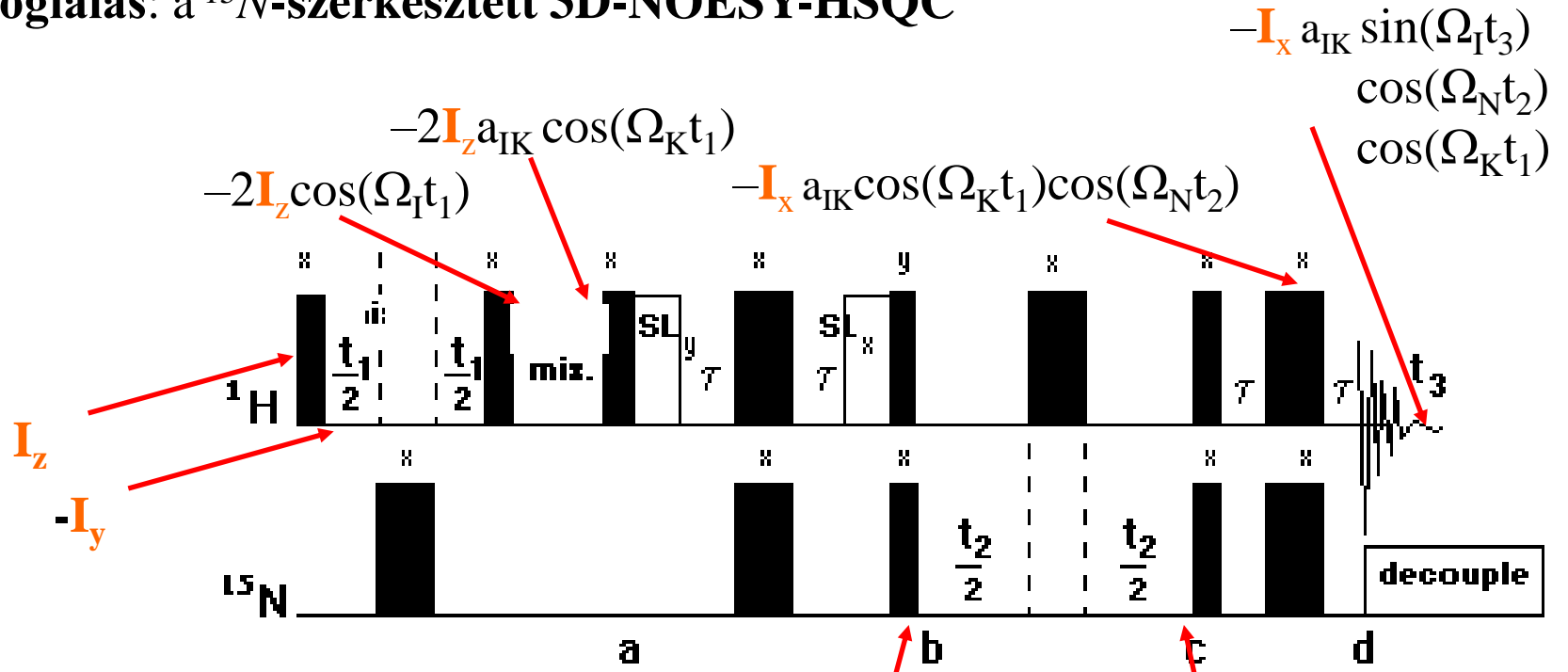
memo.9: heteronuclear spin N with chemical shift Ω_N with a coupling J_{IN} (e.g. H^{NH} and N^{NH}) provides a "normal" HSQC.

memo10: $\{^1\text{H}-^{13}\text{C}\}$ 3D NOESY-HSQC only the aliphatic carbons [$\text{C}^\alpha, \text{C}^\beta, \dots, \text{C}^\epsilon \{ \approx 40 \pm 30 \text{ ppm} \}$] and protons [$\text{H}^\alpha, \text{H}^\beta, \dots, \text{H}^\epsilon \{ \approx 3 \pm 3 \text{ ppm} \}$] are recorded.

The NOESY and the HSQC planes.

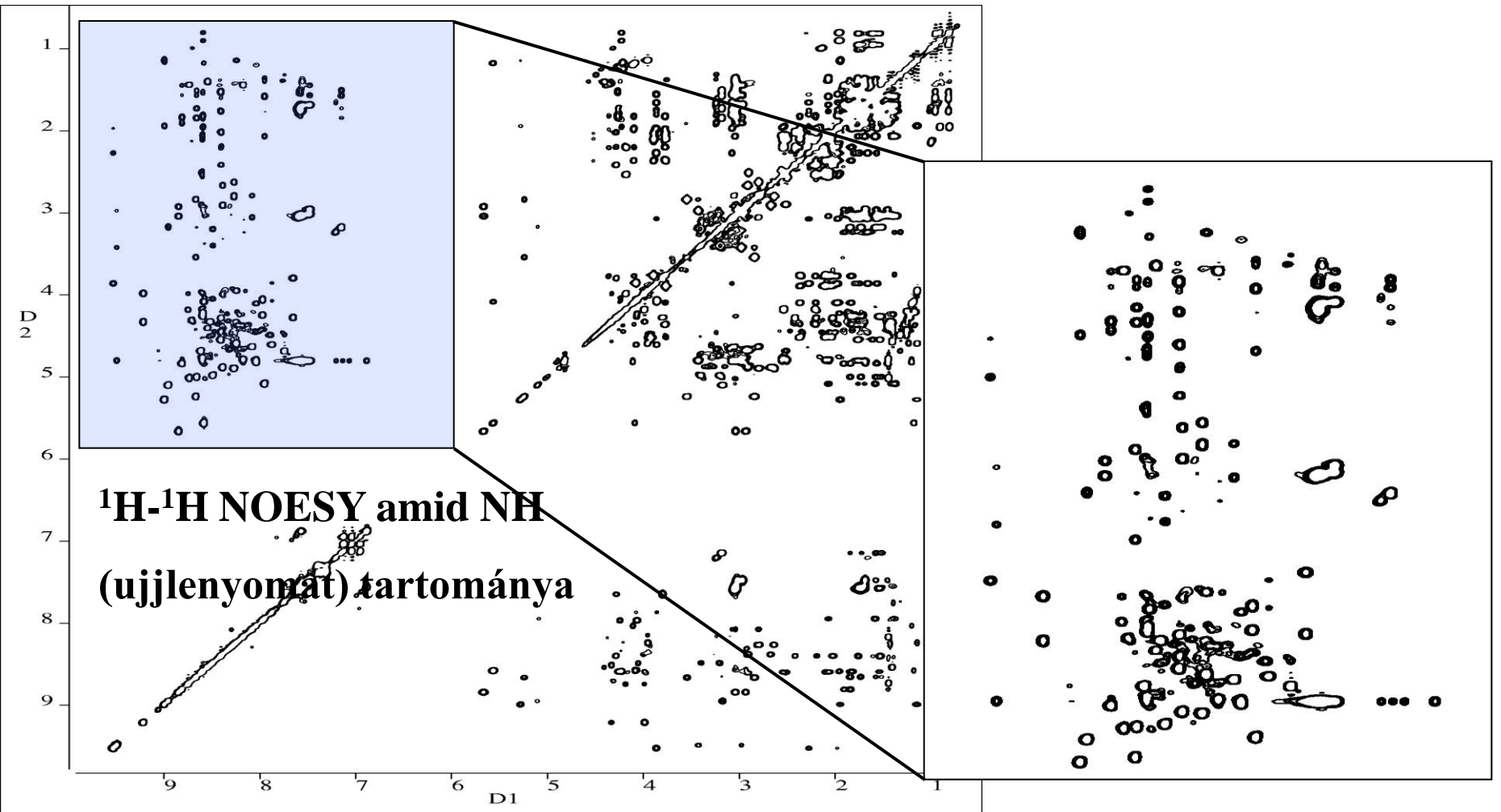


Összefoglalás: a ^{15}N -szerkesztett 3D-NOESY-HSQC



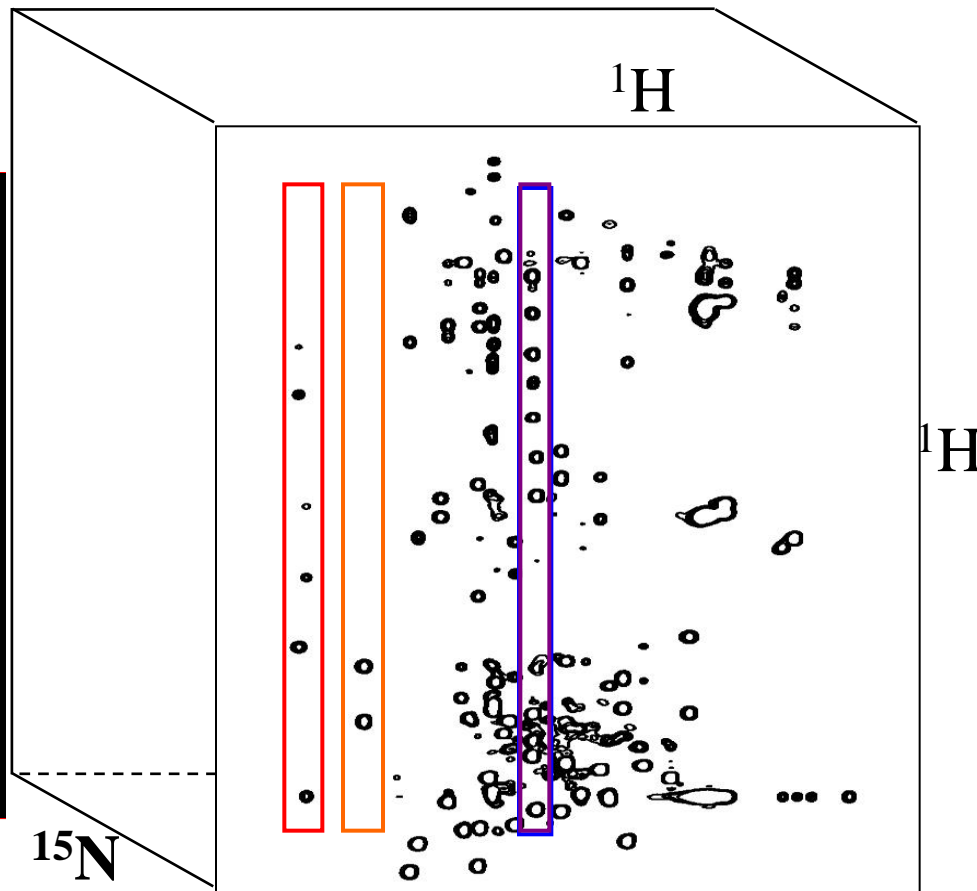
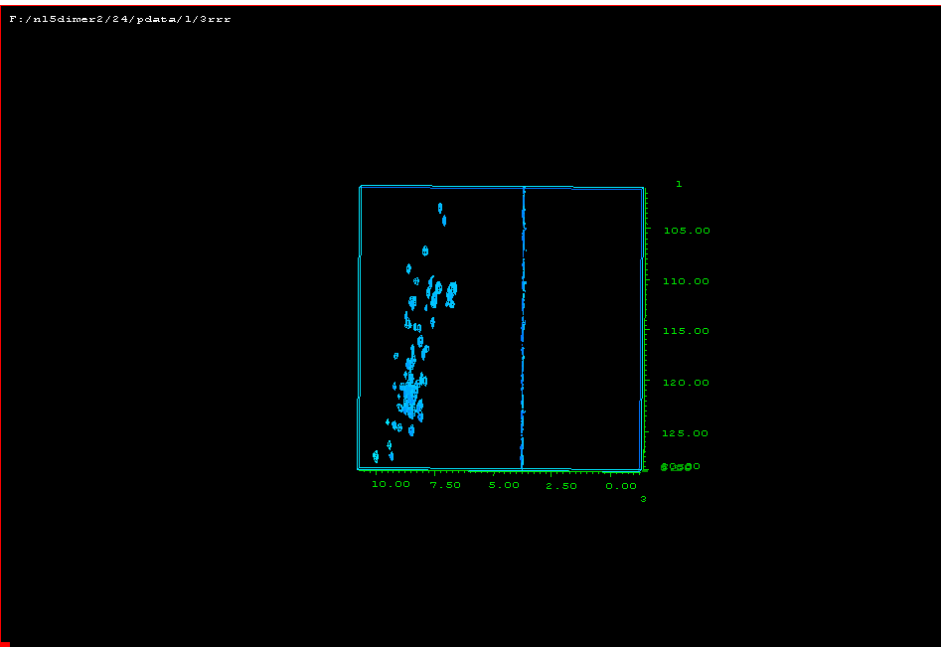
a_{IK} = off diagonális intenzitás
 A J_{IK} okozta modulációtól eltekintünk

^{15}N -szerkesztett NOESY spektrum

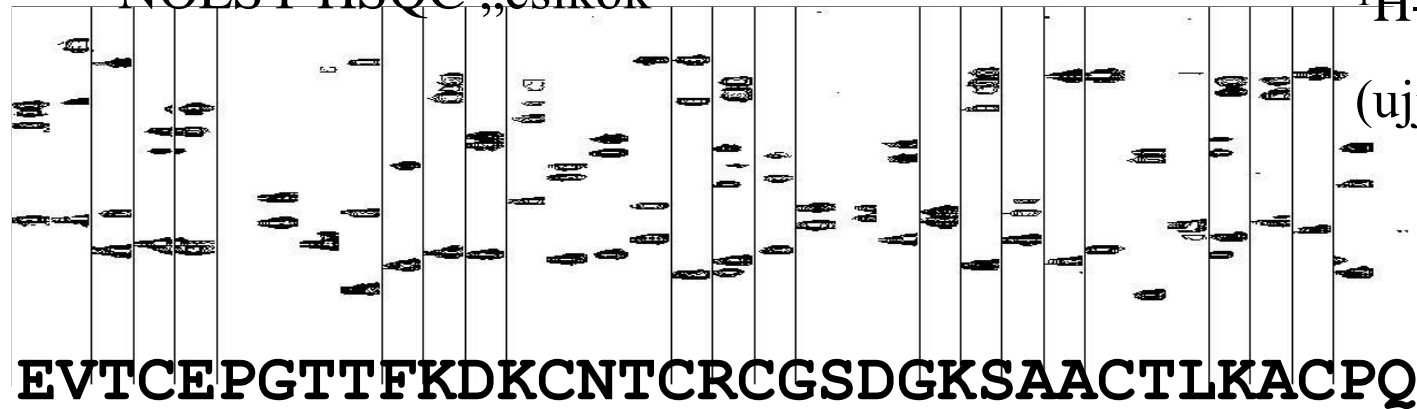


Homonukleáris 2D NOESY

^{15}N -szerkesztett 2D NOESY



NOESY-HSQC „csíkok”



^1H - ^1H TOCSYamid NH
(ujjlennyomat) tartománya

EVTCEPGTTFKDKCNTCRCGSDGKSAACTLKACPQ