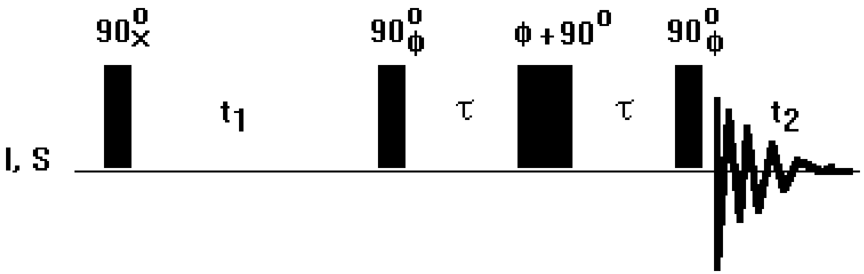
RELAY = RELAYed correlation spectroscopy

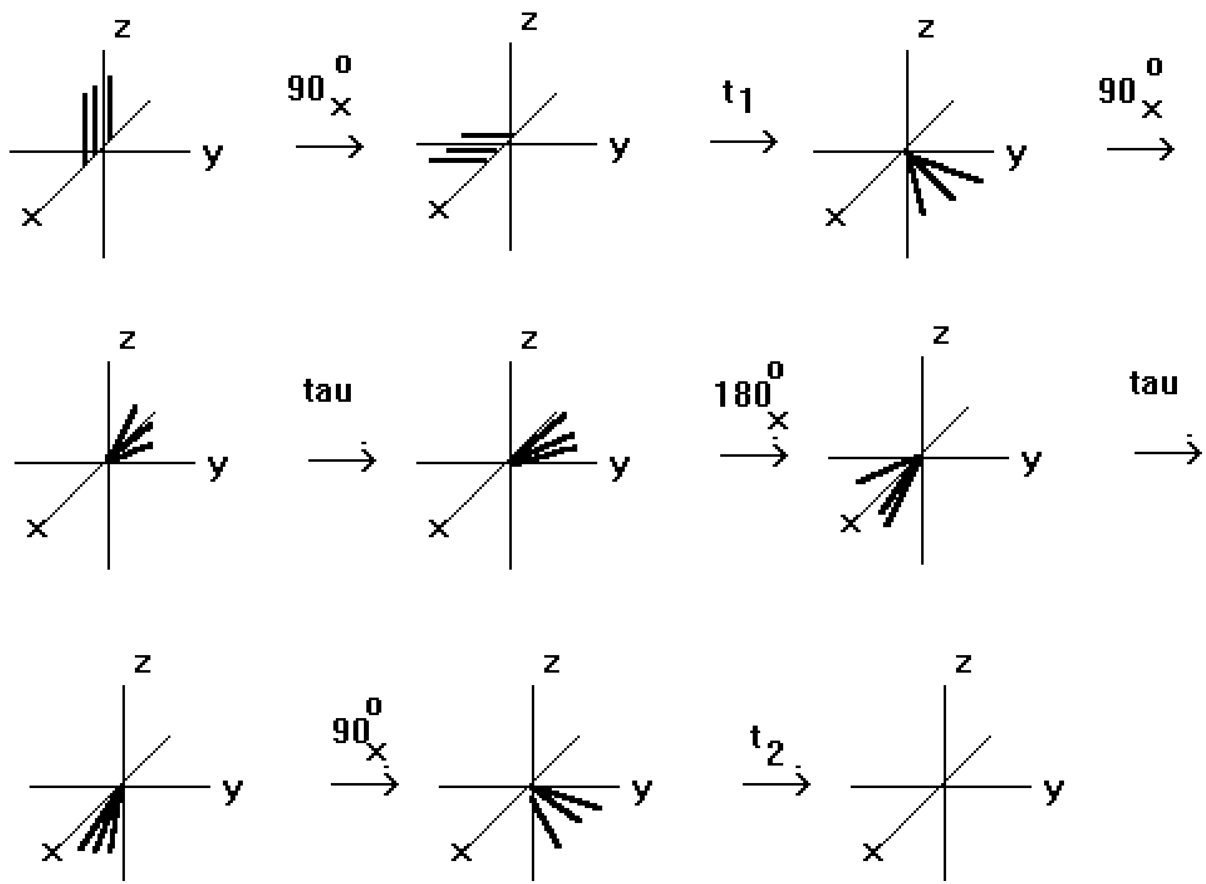
The pulse sequence: $90°_x$ - t_1 - [$90°\phi$ - τ- $(90°+\phi)$- τ- $90°\phi$]- t_2

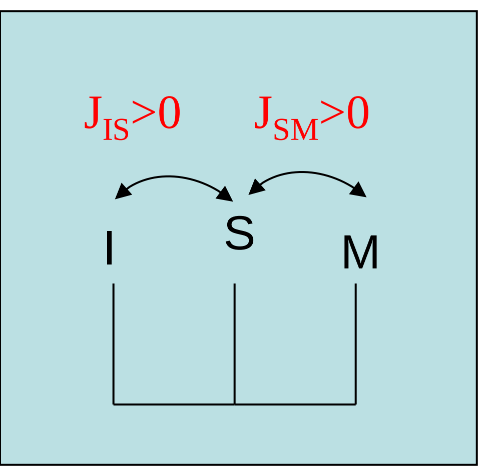


It can be phase-cycled but not essential.
If not phase-cycled then:

$90°_x$ - t_1 - $90°_x$ - τ- $180°_x$- τ- $90°_x$- t_2

Consider: Ω_I, Ω_S,Ω_M
and $J_{IS}>0$, $J_{SM}>0$
but $\mathbf{J_{IM}=0}$





σ[eq.] $\quad$ $\mathbf{I_z}$ **,** $\mathbf{M_z}$ **and** $\mathbf{S_z}$

$\hat{H} = \pi/2 \; (\hat{I}_x + M_x + \check{S}_x)$ $\quad$ $\downarrow 90°_x$

σ[0] $\quad$ $-\mathbf{I_y}$ **,** $-\mathbf{M_y}$ **and** $-\mathbf{S_y}$

$\hat{H} = \hat{I}_z(\Omega_I t_1) + M_z(\Omega_M t_1) + \check{S}_z(\Omega_S t_1)$ $\quad$ $\downarrow \; t_1$

$-\mathbf{I}_y \cos(\Omega_I t_1) + \mathbf{I}_x \sin(\Omega_I t_1)$

$-\mathbf{M}_y \cos(\Omega_M t_1) + \mathbf{M}_x \sin(\Omega_M t_1)$

$-\mathbf{S}_y \cos(\Omega_S t_1) + \mathbf{S}_x \sin(\Omega_S t_1)$

$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi t_1)$ $\quad$ $\downarrow$

$-\mathbf{I}_y \cos(\Omega_I t_1)\cos(\pi J_{IS} t_1)$

$+2\mathbf{I}_x\mathbf{S}_z \cos(\Omega_I t_1)\sin(\pi J_{IS} t_1)$

$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$

$+2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$

$-\mathbf{S}_y \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)$

$+2\mathbf{S}_x\mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)$

$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)$

$+2\mathbf{S}_y\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)$

$-\mathbf{M}_y \cos(\Omega_M t_1)$

$+\mathbf{M}_x \sin(\Omega_M t_1)$

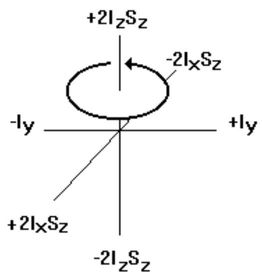
$$-\mathbf{I}_y \cos(\Omega_I t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{I}_x\mathbf{S}_z \cos(\Omega_I t_1)\sin(\pi J_{IS} t_1)$$
$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$$
$$-\mathbf{S}_y \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{S}_x\mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)$$
$$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{S}_y\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)$$
$$-\mathbf{M}_y \cos(\Omega_M t_1) + \mathbf{M}_x \sin(\Omega_M t_1)$$

$\hat{H} = 2S_zM_z(J_{SM}\pi t_1)$

$\downarrow$

$$-\mathbf{I}_y \cos(\Omega_I t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{I}_x\mathbf{S}_z \cos(\Omega_I t_1)\sin(\pi J_{IS} t_1)$$
$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1) + 2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$$

and



$$-\mathbf{S}_y \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_x\mathbf{M}_z \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_x\mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$$
$$+4\mathbf{S}_y\mathbf{M}_z\,\mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$$
$$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_y\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$$
$$+4\mathbf{S}_x\mathbf{M}_z\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$$

and

$$-\mathbf{M}_y \cos(\Omega_M t_1)\cos(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_z\mathbf{M}_x \cos(\Omega_M t_1)\sin(\pi J_{SM} t_1)$$
$$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$$
$$+2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$$

$-\mathbf{I}_y \cos(\Omega_I t_1)\cos(\pi J_{IS} t_1)$ $+2\mathbf{I}_x\mathbf{S}_z \cos(\Omega_I t_1)\sin(\pi J_{IS} t_1)$
$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$ $+2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$
$-\mathbf{S}_y \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$ $+2\mathbf{S}_x\mathbf{M}_z \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$+2\mathbf{S}_x\mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$ $+4\mathbf{S}_y\mathbf{M}_z \mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$ $+2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$+2\mathbf{S}_y\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$ $+4\mathbf{S}_x\mathbf{M}_z\mathbf{I}_z \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$-\mathbf{M}_y \cos(\Omega_M t_1)\cos(\pi J_{SM} t_1)$ $+2\mathbf{S}_z\mathbf{M}_x \cos(\Omega_M t_1)\sin(\pi J_{SM} t_1)$
$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$ $+2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$

$\hat{H} = \pi/2 \ (\hat{I}_x + M_x + \check{S}_x)$ $\downarrow 90°_x$ *memo. = cos(π/2) = 0, sin(π/2) = 1*

$-\mathbf{I}_z \cos(\Omega_I t_1)\cos(\pi J_{IS} t_1)$
$-2\mathbf{I}_x\mathbf{S}_y \cos(\Omega_I t_1)\sin(\pi J_{IS} t_1)$
$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$
$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$
$-\mathbf{S}_z \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$
$-2\mathbf{S}_x\mathbf{M}_y \cos(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$-2\mathbf{S}_x\mathbf{I}_y \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$
$+4\mathbf{S}_z\mathbf{M}_z \mathbf{I}_z \cos(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$
$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$
$+4\mathbf{S}_x\mathbf{M}_y\mathbf{I}_y \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$
$-\mathbf{M}_z \cos(\Omega_M t_1)\cos(\pi J_{SM} t_1)$
$-2\mathbf{S}_y\mathbf{M}_x \cos(\Omega_M t_1)\sin(\pi J_{SM} t_1)$
$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$
$\sigma[t_1,0]$ $-2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$

Among these 16 terms only the following 7 describe single quantum coherences

Among these 16 terms only the following 7 describe single quantum coherences

in-phase I
anti-phase S
in-phase S
anti-phase M
anti-phase I
in-phase M
anti-phase S

$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$ ↲

$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$ ↲

$+\mathbf{S}_x \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$

$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_S t_1)\cos(\pi J_{IS} t_1)\sin(\pi J_{SM} t_1)$

$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_S t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{SM} t_1)$

$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$ ↲

$-2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$ ↲

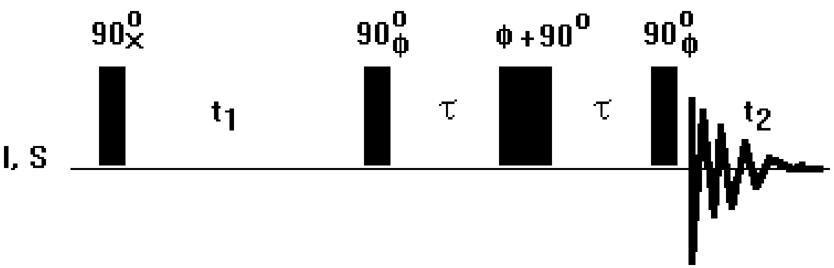
As we are interested in transfer from **I to M *via* S** and thus we keep only the **in-phase and the anti-phase I and M** coherences:

$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$

$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$

$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$

$-2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$



During the [$90°_\phi$ **- τ- (90°+ϕ)- τ-** $90°_\phi$], whit an echo inserted, **only coupling** is effective while chemical shift is refocused.

if $\phi = x$, and the effective pulse is [$90°_x$ - τ- $180°_x$- τ- $90°_x$] then only the following two terms evolve:

A: $-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$

B: $-2\mathbf{S}_y\mathbf{M}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$.

memo: the "echo time" is 2τ

A: the first term evolves during $\mathbf{2\tau}$ such as:

$$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi\mathbf{2\tau})$$

$$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)$$

$$\downarrow$$

$$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{IS} 2\tau)$$
$$+\mathbf{S}_x \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS} 2\tau)$$

$$\hat{H} = 2\check{S}_z M_z(J_{SM}\pi\mathbf{2\tau})$$

$$\downarrow$$

$$-2\mathbf{I}_z\mathbf{S}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{IS} 2\tau)\cos(\pi J_{SM} 2\tau)$$
$$+4\mathbf{I}_z\mathbf{S}_x\mathbf{M}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\cos(\pi J_{IS} 2\tau)\sin(\pi J_{SM} 2\tau)$$
$$+\mathbf{S}_x \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS} 2\tau)\cos(\pi J_{SM} 2\tau)$$
$$-2\mathbf{M}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS} 2\tau)\sin(\pi J_{SM} 2\tau)$$ ☻

$-2M_y\mathbf{S}_z$ is an anti-phased magnetization on M; a term originating from **I** and *via* S is now transfer. to **M**.

B: the second term under the same condition evolves such as :

$$-2\mathbf{M}_z\mathbf{S}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)$$

$$\hat{H} = 2\check{S}_z M_z(J_{SM}\pi\mathbf{2\tau})$$

$$\downarrow$$

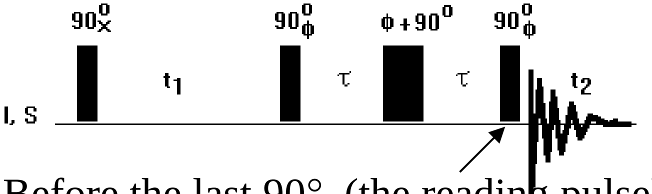
$$-2\mathbf{M}_z\mathbf{S}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\cos(\pi J_{SM} 2\tau)$$
$$+\mathbf{S}_x \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)$$

$$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi\mathbf{2\tau})$$

$$\downarrow$$

$$-2\mathbf{M}_z\mathbf{S}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\cos(\pi J_{SM} 2\tau)\cos(\pi J_{IS} 2\tau)$$
$$+4\mathbf{M}_z\mathbf{S}_x\mathbf{I}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\cos(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)$$
$$+\mathbf{S}_x \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\cos(\pi J_{IS} 2\tau)$$
$$-2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)$$ ☻

$-2\mathbf{I}_y\mathbf{S}_z$ is an anti-phased magnetization on I; a term originating from **M** and *via* S is now transfer. to **I**.



Before the last $90°_x$ (the reading pulse) beside the two unchanged terms:

$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$$
$$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1),$$

The evolution of these two is important:

$$-2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau) \text{ and}$$
$$-2\mathbf{M}_y\mathbf{S}_z \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS} 2\tau)\sin(\pi J_{SM} 2\tau).$$

$\hat{H} = \pi/2\ \ (\hat{I}_x + M_x + \check{S}_x)$

$\downarrow 90°_x$ *memo. = cos(π/2) = 0, sin(π/2) = 1*

During the reading pulse:

$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$$
$$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$$
$$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)$$
$$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS} 2\tau)\sin(\pi J_{SM} 2\tau)$$

During ACQ.

Hem = $\hat{I}_z(\Omega_I t_2)$

$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)$$
$$\downarrow$$
$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\cos(\Omega_I t_2) \quad +\mathbf{I}_y \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\sin(\Omega_I t_2)$$

$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi t_2)$

$$\downarrow$$
$$+\boldsymbol{I}_x\, sin(\Omega_I t_1)cos(\pi J_{IS} t_1)cos(\Omega_I t_2)cos(\pi J_{IS} t_2) \quad ☻$$
$$+2\mathbf{I}_y\mathbf{S}_z \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\cos(\Omega_I t_2)\sin(\pi J_{IS} t_2)$$
$$+\mathbf{I}_y \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\sin(\Omega_I t_2)\cos(\pi J_{IS} t_2)$$
$$-2\mathbf{I}_x\mathbf{S}_z \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\sin(\Omega_I t_2)\sin(\pi J_{IS} t_2)$$

During ACQ.

Hem = $M_z(\Omega_M t_2)$

$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)$

$\downarrow$

$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\cos(\Omega_M t_2)$ $\quad$ $+\mathbf{M}_y \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\sin(\Omega_M t_2)$

$\hat{H} = 2\check{S}_z M_z (J_{SM}\pi t_2)$

$\downarrow$

$+\boldsymbol{M}_x\, sin(\Omega_M t_1)cos(\pi J_{SM} t_1)cos(\Omega_M t_2)cos(\pi J_{SM} t_2)$ ☻

$+2\mathbf{M}_y\mathbf{S}_z \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\cos(\Omega_M t_2)\sin(\pi J_{SM} t_2)$

$+\mathbf{M}_y \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\sin(\Omega_M t_2)\cos(\pi J_{SM} t_2)$

$-2\mathbf{M}_x\mathbf{S}_z \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\sin(\Omega_M t_2)\sin(\pi J_{SM} t_2)$

During ACQ.

Hem = $I_z(\Omega_I t_2)$

$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)$

$\downarrow$

$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)\cos(\Omega_I t_2)$

$+2\mathbf{S}_z\mathbf{I}_x \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)\sin(\Omega_I t_2)$

$\hat{H} = 2\hat{I}_z\check{S}_z(J_{IS}\pi t_2)$

$\downarrow$

$-2\mathbf{S}_z\mathbf{I}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)\cos(\Omega_I t_2)\cos(\pi J_{IS} t_2)$

$+\boldsymbol{I}_x\, sin(\Omega_M t_1)sin(\pi J_{SM} t_1)sin(\pi J_{SM} 2\tau)sin(\pi J_{IS} 2\tau)cos(\Omega_I t_2)sin(\pi J_{IS} t_2)$ ☻

$+2\mathbf{S}_z\mathbf{I}_x \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)\sin(\Omega_I t_2)\cos(\pi J_{IS} t_2)$

$+\mathbf{I}_y \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)\sin(\Omega_I t_2)\sin(\pi J_{IS} t_2)$

During ACQ.

Hem = $M_z(\Omega_M t_2)$

$$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)$$

$$\downarrow$$

$$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\cos(\Omega_M t_2)$$
$$+2\mathbf{S}_z\mathbf{M}_x \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\sin(\Omega_M t_2)$$

$\hat{H} = 2M_z\check{S}_z(J_{SM}\pi t_2)$

$$\downarrow$$

$$-2\mathbf{S}_z\mathbf{M}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\cos(\Omega_M t_2)\cos(\pi J_{SM} t_2)$$
$$+\boldsymbol{M}_x \mathit{\sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)}\sin(\pi J_{SM}2\tau)\mathit{\cos(\Omega_M t_2)\sin(\pi J_{SM} t_2)}$$ ☻
$$+2\mathbf{S}_z\mathbf{M}_x \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\sin(\Omega_M t_2)\cos(\pi J_{SM} t_2)$$
$$+\mathbf{M}_y \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\sin(\Omega_M t_2)\sin(\pi J_{SM} t_2)$$

memo 1: receiver on x

therefore only the four x term remain:

$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\cos(\Omega_I t_2)\cos(\pi J_{IS} t_2)$$
$$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\cos(\Omega_M t_2)\cos(\pi J_{SM} t_2)$$
$$+\mathbf{I}_x \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\sin(\pi J_{SM}2\tau)\sin(\pi J_{IS}2\tau)\cos(\Omega_I t_2)\sin(\pi J_{IS} t_2)$$
$$+\mathbf{M}_x \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\sin(\pi J_{IS}2\tau)\sin(\pi J_{SM}2\tau)\cos(\Omega_M t_2)\sin(\pi J_{SM} t_2)$$

The term $\alpha = \sin(\pi J_{SM}2\tau)\sin(\pi J_{IS}2\tau)$ is the transfer function between I and M. Now:

$$+\mathbf{I}_x \sin(\Omega_I t_1)\cos(\pi J_{IS} t_1)\cos(\Omega_I t_2)\cos(\pi J_{IS} t_2)$$
$$+\mathbf{M}_x \sin(\Omega_M t_1)\cos(\pi J_{SM} t_1)\cos(\Omega_M t_2)\cos(\pi J_{SM} t_2)$$
$$+\mathbf{I}_x\boldsymbol{\alpha} \sin(\Omega_M t_1)\sin(\pi J_{SM} t_1)\cos(\Omega_I t_2)\sin(\pi J_{IS} t_2)$$
$$+\mathbf{M}_x\boldsymbol{\alpha} \sin(\Omega_I t_1)\sin(\pi J_{IS} t_1)\cos(\Omega_M t_2)\sin(\pi J_{SM} t_2)$$

memo 2: $\sin(A)\cos(B) = 1/2[\sin(A+B)+\sin(A-B)]$
$\cos(A)\sin(B) = 1/2[\sin(A+B)-\sin(A-B)]$
$\sin(A)\sin(B) = 1/2[\cos(A-B)-\cos(A+B)]$
$\cos(A)\cos(B) = 1/2[\cos(A+B)+\cos(A-B)]$

therefore:

$+1/4\mathbf{I}_x \; [+\sin\{(\Omega_I + \pi J_{IS})t_1\} + \sin\{(\Omega_I - \pi J_{IS})t_1\}][+\cos\{(\Omega_I + \pi J_{IS})t_2\} + \cos\{(\Omega_I - \pi J_{IS})t_2\}]$

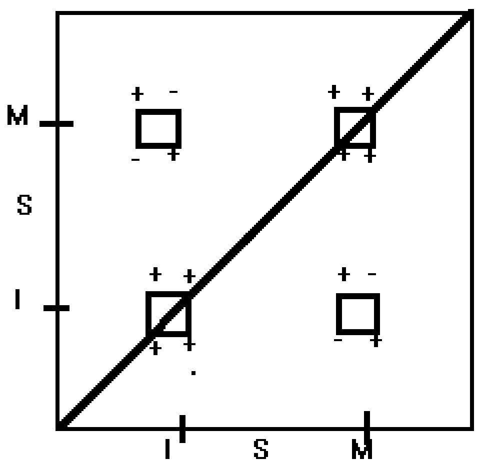
$+1/4\mathbf{M}_x \; [+\sin\{(\Omega_M + \pi J_{SM})t_1\} + \sin\{(\Omega_M - \pi J_{SM})t_1\}][+\cos\{(\Omega_M + \pi J_{SM})t_2\} + \cos\{(\Omega_M - \pi_{SM})t_2\}]$

$+1/4\,\mathbf{I}_x\,\alpha \; [+\cos\{(\Omega_M - \pi J_{SM})t_1\} - \cos\{(\Omega_M + \pi J_{SM})t_1\}][+\sin\{(\Omega_I + \pi J_{IS})t_2\} - \sin\{(\Omega_I - \pi J_{IS})t_2\}]$

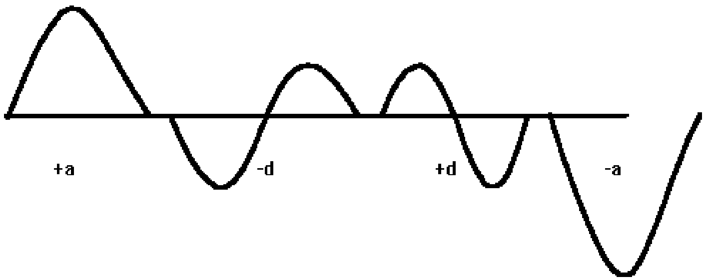
$+1/4\mathbf{M}_x\alpha[+\cos\{(\Omega_I - \pi J_{IS}t_1\} - \cos\{(\Omega_I + \pi J_{IS})t_1\}][+\sin\{(\Omega_M + \pi J_{SM})t_2\} - \sin\{(\Omega_M - \pi J_{SM})t_2\}]$

the following terms can be found

$\mathbf{I}_x$ [+ .. + .. + .. + ..] at Ω_I , Ω_I
$\mathbf{M}_x$[+ .. + .. + .. + ..] at Ω_M , Ω_M
$\mathbf{I}_x$ [+ .. - .. + .. - ..] at Ω_M , Ω_I
$\mathbf{M}_x$[+ .. - .. + .. - ..] at Ω_I , Ω_M



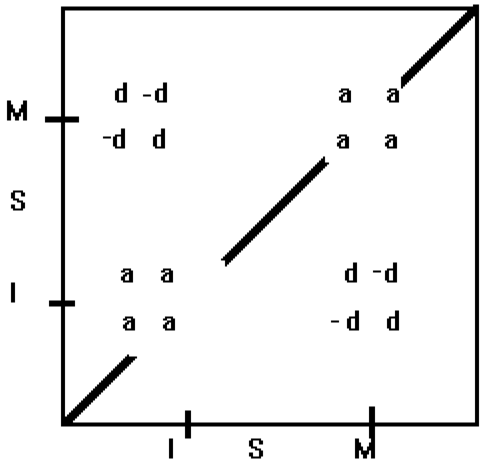
memo: line shapes
positive absorptive (+a),
negative absorptive (-a),
positive dispersive (+d),
negative dispersive (-d),



if one sets the phase that *sin is absorptive (a) in t_1*
cos is absorptive (d) in t_2

$\mathbf{I}_x$ [+a .. +a .. +a .. +a ..] at Ω_I , Ω_I
$\mathbf{M}_x$[+a .. +a.. +a .. +a ..] at Ω_M , Ω_M
$\mathbf{I}_x$ [+d .. –d .. +d .. –d .] at Ω_M , Ω_I
$\mathbf{M}_x$[+d .. –d .. +d .. –d .] at Ω_I , Ω_M

so the diagonals have absorptive and all off-diagonals have dispersive line shape. As expected for a COSY type spectrum.



The term $\alpha = \sin(\pi J_{SM} 2\tau)\sin(\pi J_{IS} 2\tau)$ term can be maximised if
$\tau = 1/(4J_{IS})$ as well as $\tau = 1/(4J_{SM})$.
but in general $J_{IS} \neq J_{SM}$